



Ensuring Conservatism of the Equivalent Test in the Damage Potential Method

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The Damage Potential Method Reduces Conservatism in Flight Data Analysis



- **Typical flight data analysis assumes that each component sees the maximum 1-second level throughout a large part of flight**
 - **Flight is nonstationary; max level occurs for only a few seconds**
 - **This assumption is therefore very conservative**

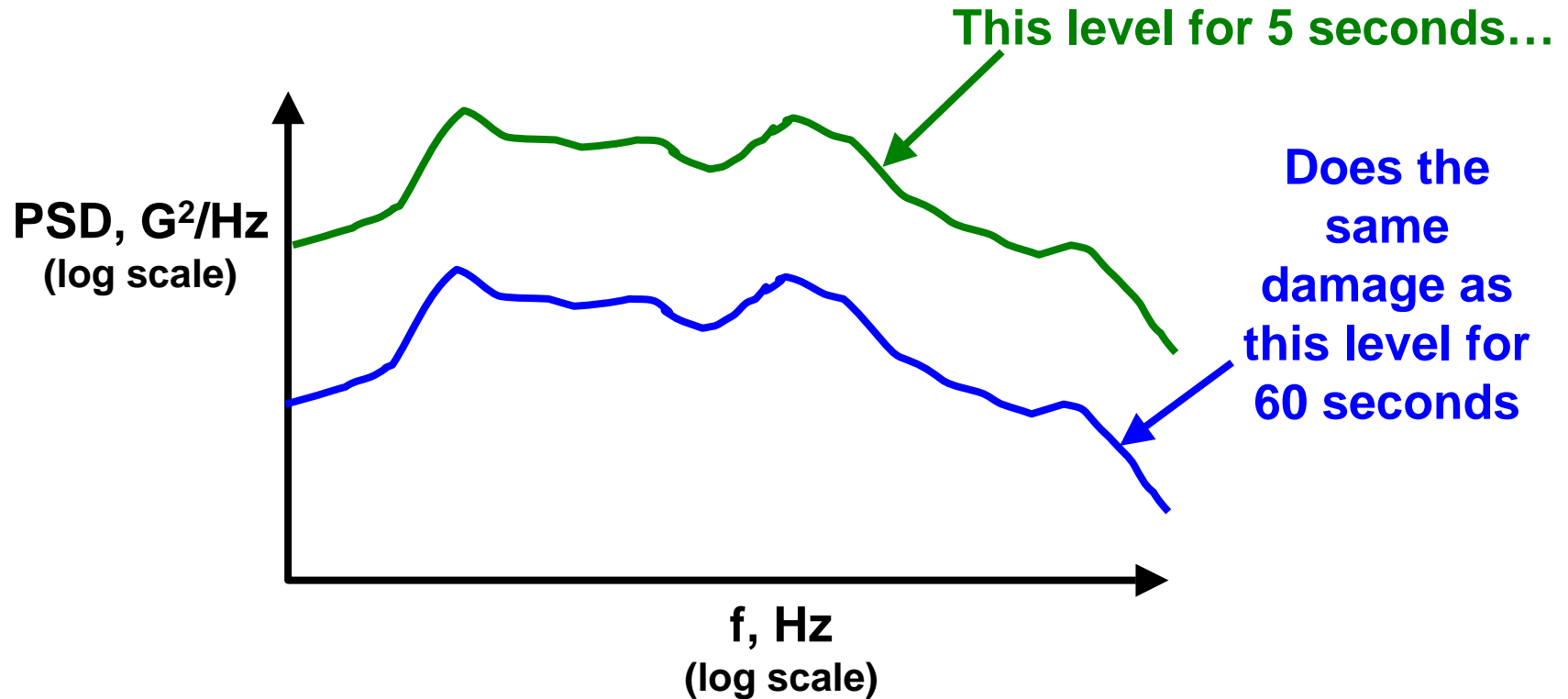
- **This conservatism in typical flight data analysis may cause components to require re-qualification**
 - **A less conservative analysis may show that these components are still qualified for flight**

- **To reduce conservatism in flight data analysis, the Damage Potential Method was developed by Rubin, et. al.**
 - **Method creates an equivalent stationary process that does the same or more damage to the component as (nonstationary) flight**

Equivalent Stationary Levels



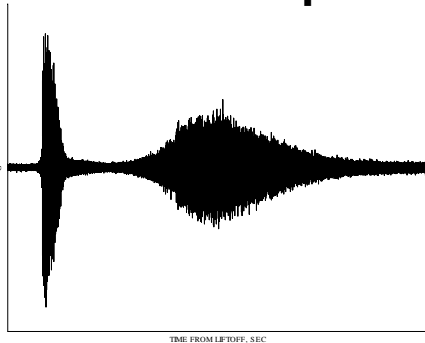
- Damage is dependent on response cycle amplitudes
 - Function of environment's level and duration
 - One can be traded for the other!



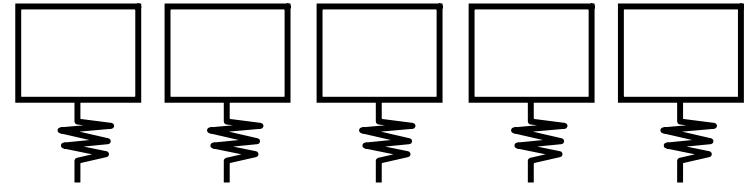
Overview of the Damage Potential Method



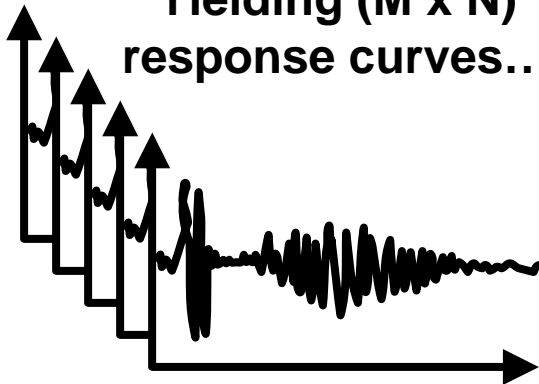
Data from a flight is used as base input to...



M x N Oscillators...
(M values of Q, N natural frequencies)



Yielding (M x N) response curves...



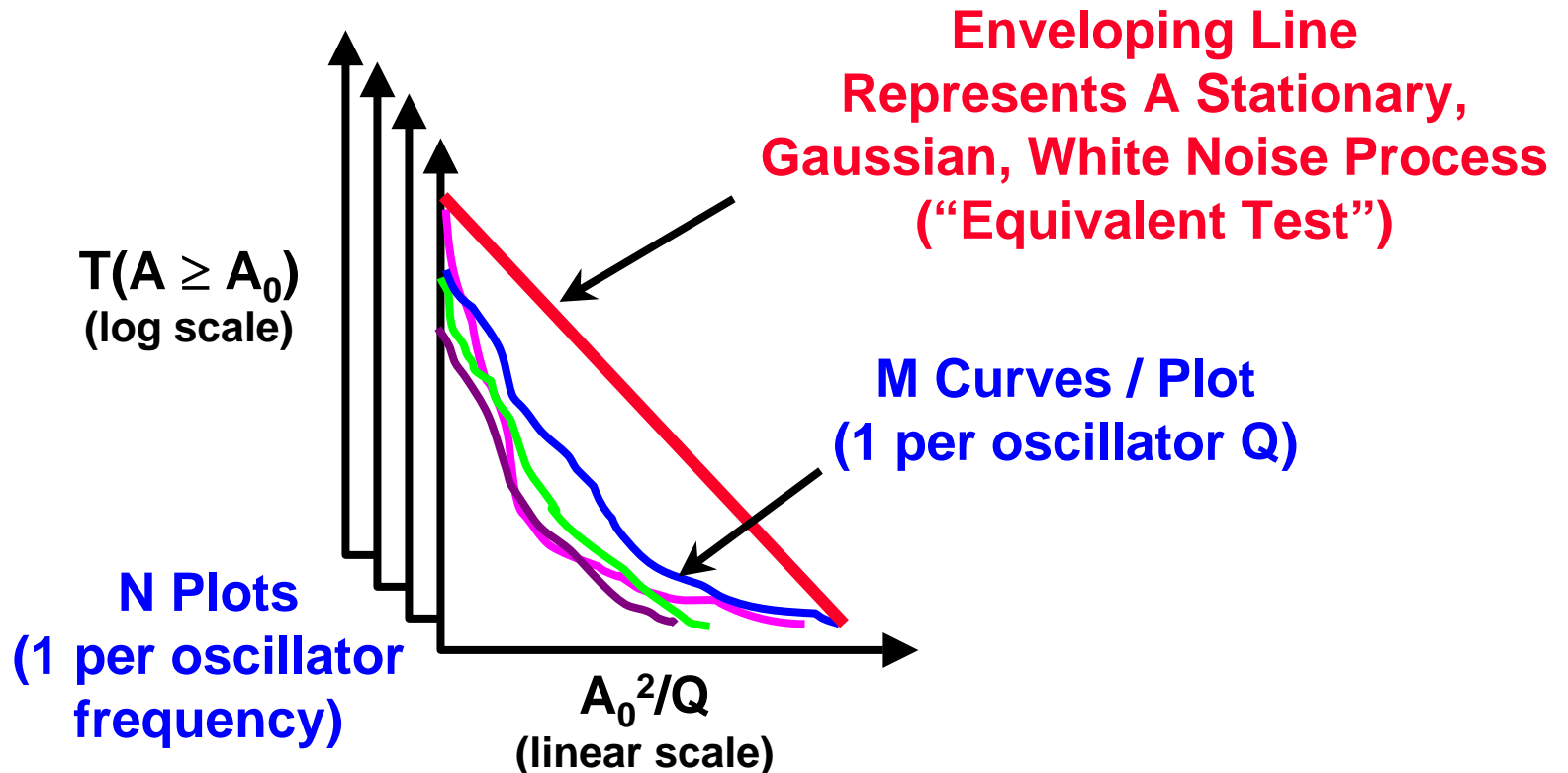
That are modified sinusoids, with near-constant frequencies but varying cycle amplitudes.

The response cycle amplitudes for each of the (M x N) curves are sorted from high to low.

Overview of the Damage Potential Method



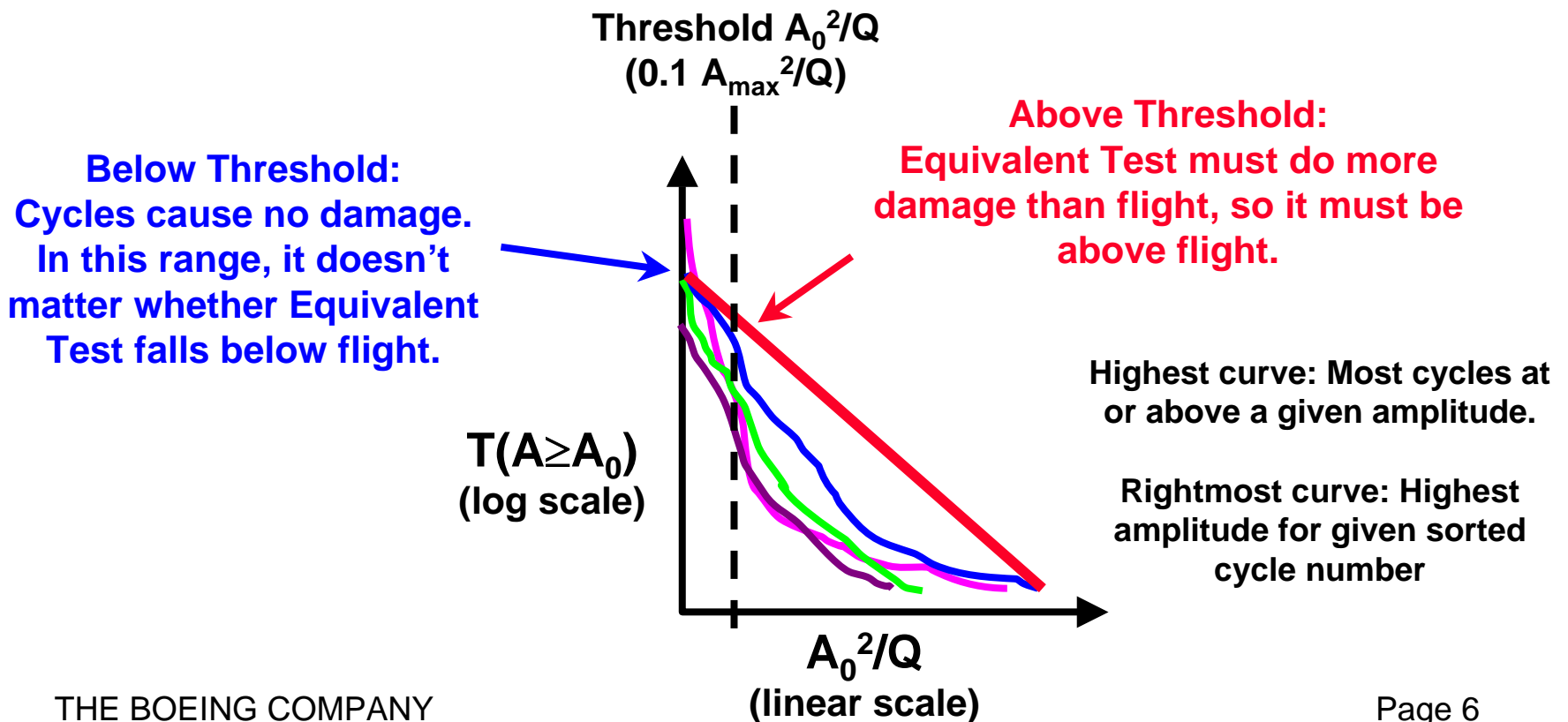
For each flight, the (M x N) sorted amplitude sets are plotted, and an enveloping line is drawn for each frequency:



Conservatism of the Equivalent Test



If the Equivalent Test is above / to the right of the flight curves above a threshold amplitude, it's conservative.



Original Formulation of Equivalent Test



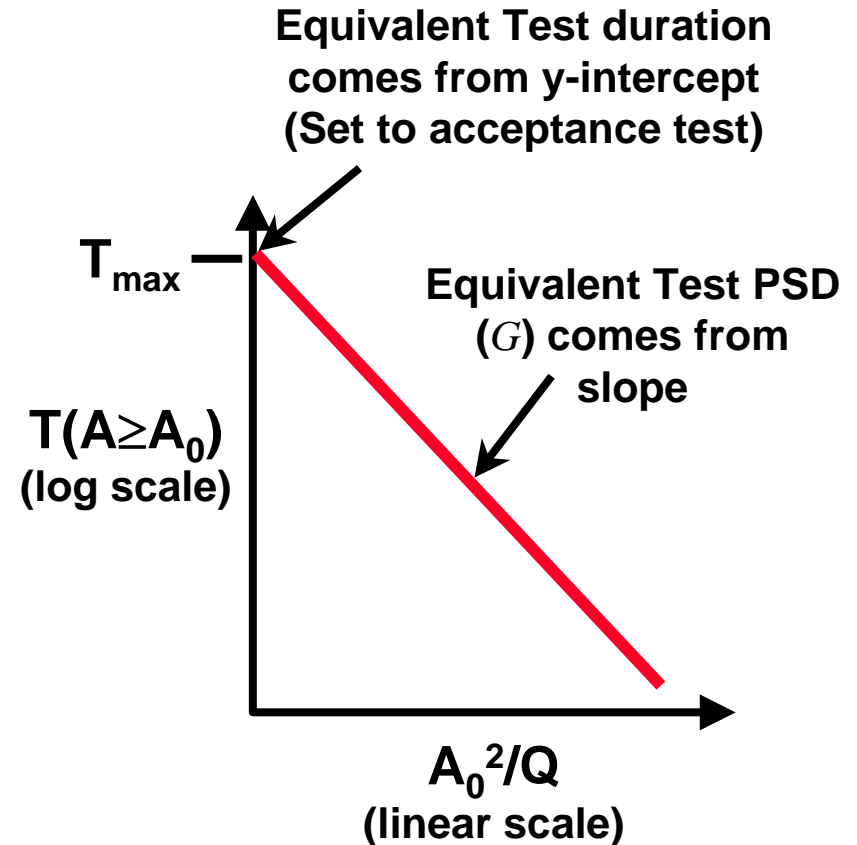
- Equivalent Test is stationary, white noise, Gaussian
- Response amplitudes are independent, Rayleigh-distributed:

$$P(A \geq A_0) = \exp\left(\frac{-A_0^2}{\pi G f_n Q}\right)$$

- So the time spent in cycles with amplitudes over A_0 is:

$$T(A \geq A_0) = T_{\max} \exp\left(\frac{-A_0^2}{\pi G f_n Q}\right)$$

which is a line on the log-linear axes shown



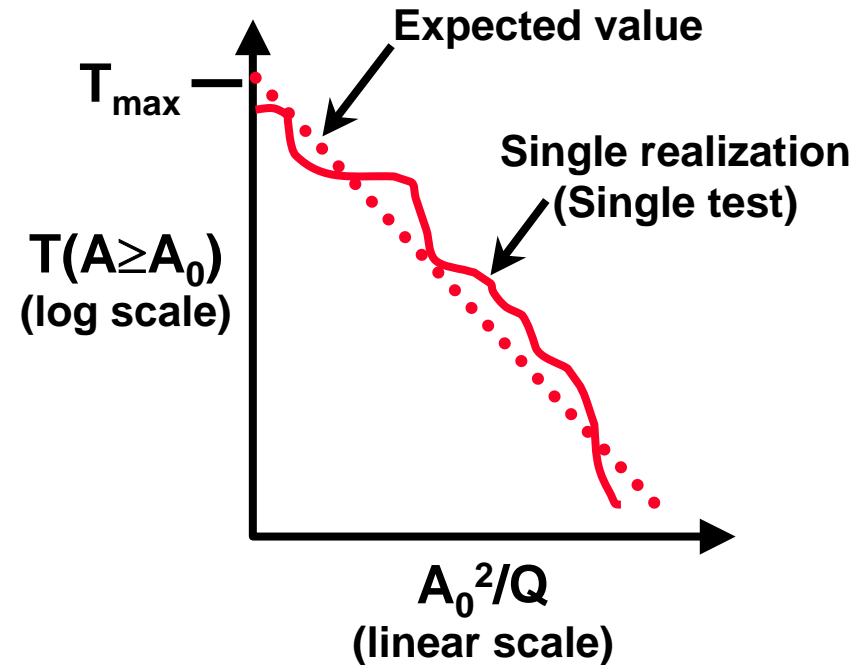


Original Formulation of Equivalent Test

- However, the red-bordered equation should be expressed in terms of expected value

$$E[T(A \geq A_0)] = T_{\max} \exp\left(\frac{-A_0^2}{\pi G f_n Q}\right)$$

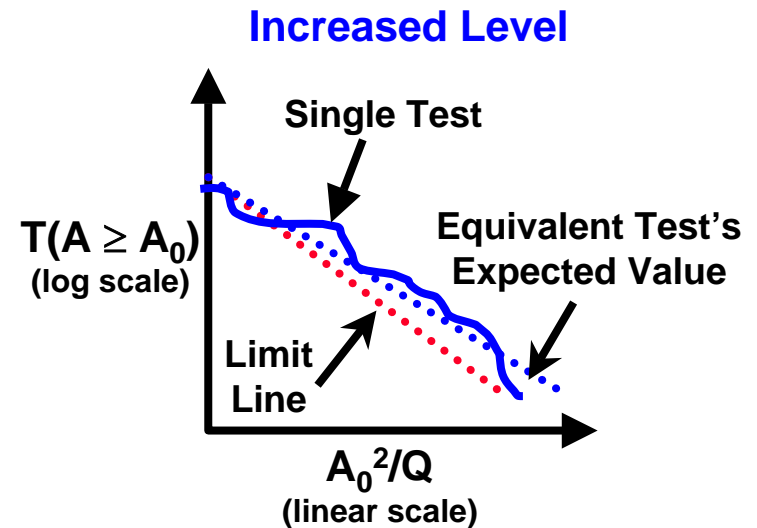
- Equivalent Test is conservative when a single realization's ("Single Test") curve envelopes flight
 - Single test's curve will deviate from the expected line
 - Non-conservatism possible if expected-value line is close to the flight curves





Revised Formulation of Equivalent Test

- We want to ensure that single test curves don't drop below flight data
 - Limit line (orig. equivalent test) is at or above flight data
 - Ensure that single test curves stay above limit line!
- To do this, we can increase the Equivalent Test's level above limit line
 - Single test curves will be centered on higher lines
 - The higher the level, the higher the probability that any single test curve stays above the limit line



Revised Formulation of Equivalent Test



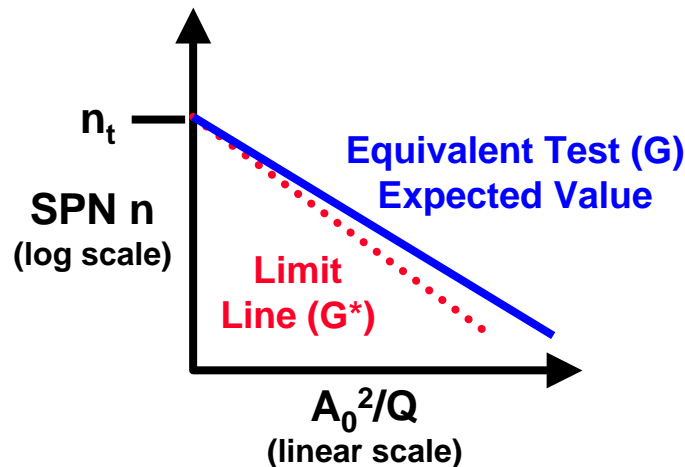
- **How much should the level of the equivalent test be increased from the limit line to ensure a given probability of single-test curve conservatism?**

- **Answering this requires probabilistic analysis**
 - **Use sorted response cycle number (or sorted peak number, SPN) n instead of $T(A \geq A_0)$**
 - **No bins used**
 - **Equivalent Test is a Gaussian, stationary, white noise process**
 - **Response cycle amplitudes are independent Rayleigh random variables**

Probability of Single Test Curve Exceeding Given Point on Limit Line



- First, I examined the probability of a single test curve exceeding a given point on the limit line, which depends on:
 - Total number of response cycles n_t
 - Amount of PSD level increase, G/G^*
 - Sorted peak number n (y-value) of the point on the limit line being analyzed

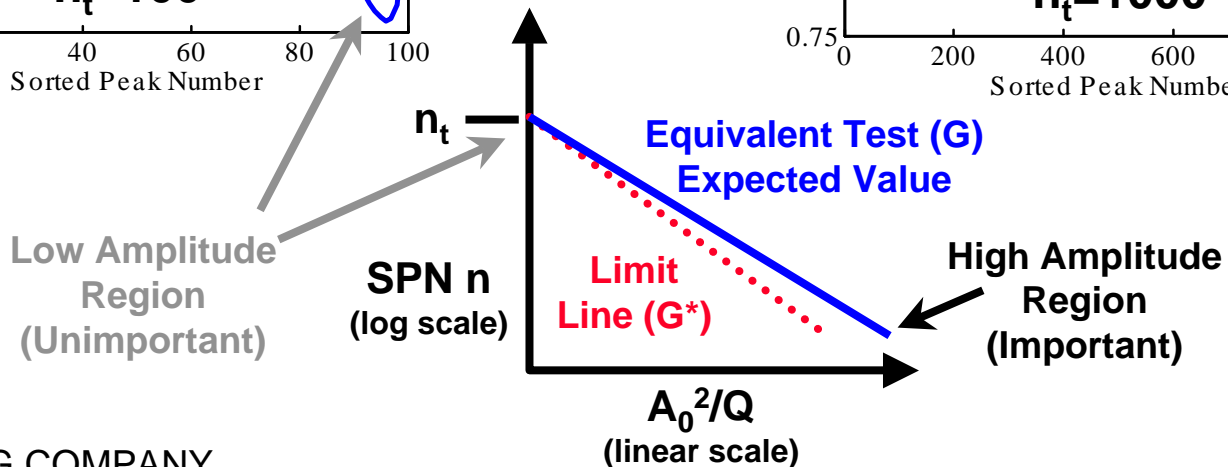
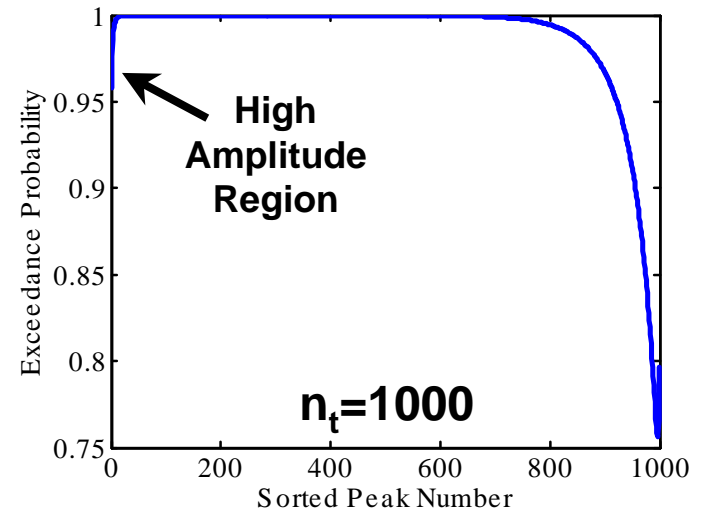
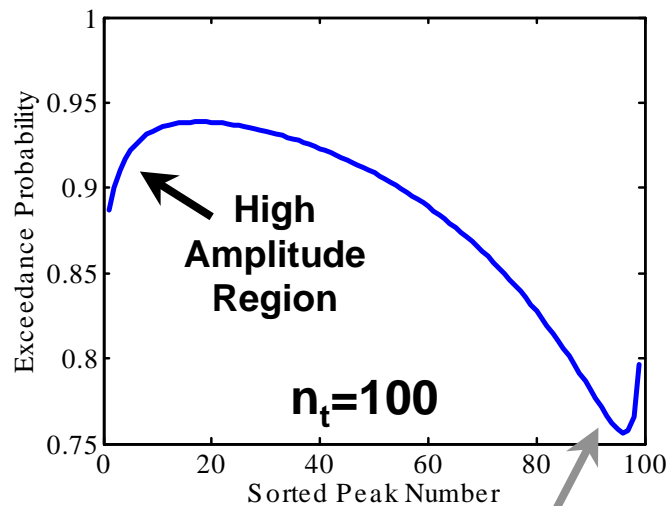


Mathematics used for this analysis are in attached text.

Probability of Single Test Curve Exceeding Given Point on Limit Line



Level increase: For $G/G^* = 1.2$ (+0.8 dB), $n_t=100, 1000$, the single-point limit line exceedance probabilities are:



Probability of Single Test Curve Exceeding Region of Limit Line



- The preceding work only dealt with the probability of single test exceedance (conservatism) at a single point on the limit line
 - It cannot be used to derive the probability of single test exceedance over a region of the limit line (i.e., above some threshold A)
- A full probabilistic description of the problem was done, but the resulting probability series did not converge even for $n_t=100$
- A probabilistic description of the problem, focusing on the high-amplitude region, does converge for large n_t
 - The problem is partitioned into a complete set of distinct events, each of which has a calculable probability

Probability of Single Test Curve Exceeding Region of Limit Line



- The probabilities of the distinct failure modes, F_n , are calculated
 - F_n = probability that single test exceeded highest $n-1$ amplitudes on the limit line, but not the n^{th} highest
 - Details in attached text

- Failure series convergence criterion: $F_n < 0.001 \sum F_k$
 - If convergence not seen by term 15, that case is discarded

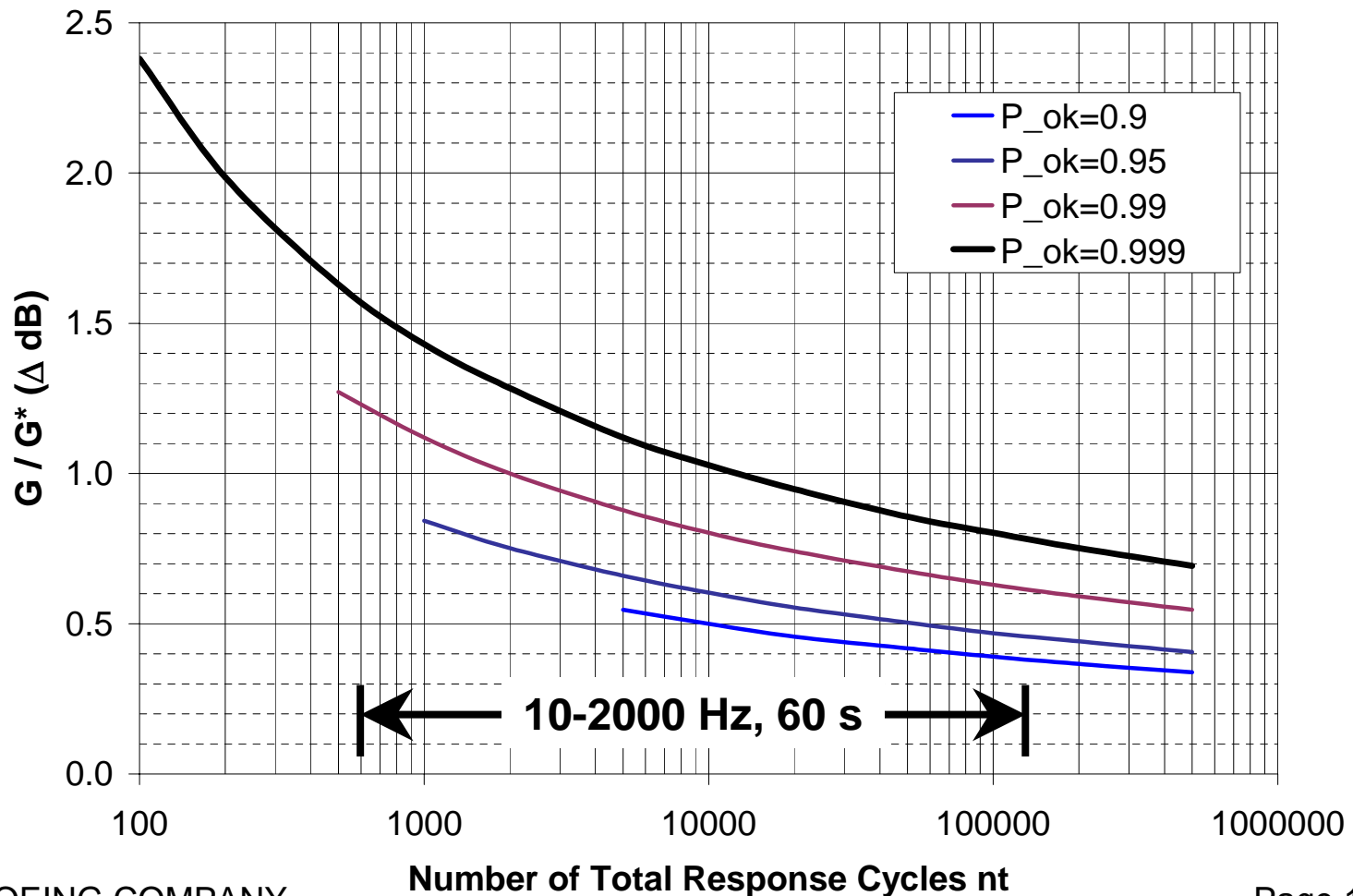
- Probability of single test conservatism: $P_{ok} = 1 - \sum F_k$

- Probability that single test exceeds limit line can be calculated as function of level increase
 - Plotted as level increase needed for given probability that any given single test is conservative!

Amount to Raise Equivalent Test Level Above Original Formulation



PSD Increase (dB) to Ensure Single Test Curve Exceeds Limit Line





The Revised Damage Potential Method

- For each set of flight data, generate the set of $T(A \geq A_0)$ vs. A_0^2/Q plots, and draw a limit line for each plot
 - Ensure that all limit lines have the y-intercept T_{\max} at the acceptance test duration
 - Obtain the PSD level (G) for each plot's limit line
- Increase the PSD levels to form Equivalent Test levels with a given (single realization) conservatism probability P_{ok}
 - Increase level at each frequency according to the chart, where $n_t = f_n T_{\max}$
 - Use the Equivalent Test's level at each frequency to form an equivalent stationary spectrum
- Compute P95/50 of Equivalent Test spectra across flights (or otherwise account for flight-to-flight scatter) and compare to qual

Conclusions



- **The Damage Potential Method reduces conservatism in flight data analysis**
 - **May remove the need to re-qualify some components due to flight data**

- **The original formulation of the flight data's equivalent stationary process ("Equivalent Test") may cause non-conservatism in the method**

- **Using a somewhat higher level over that in the original formulation can ensure conservatism of the method**
 - **Amount to raise level to ensure given probability of conservatism can be obtained from included chart**