Sticking Thrusters into Asteroids from Permanent Bases at L1 and L3

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In this paper we propose to stick many powerful thrusters into the surface of Near Earth Asteroids (NEAs) in order to deflect them at will either:

1) Away from the Earth, or

2) Into elliptical orbits within the Earth's sphere of influence, so that they can later be mined at will.

3) In addition, we propose to create a system of two permanent space bases at the distance of the Moon and located exactly at the two Lagrangian Points L1 and L3 of the Earth-Moon system in order to house the many thrusters ready to be shot against nearby passing NEAs.

4) This might be done, for instance in April 2029 to slow down Asteroid 99142 Apophis and making it become a satellite of the Earth, thus averting probable later collisions with the Earth itself.

In two previous published papers ([1]Maccone and [2]Maccone), this author had already proposed a system of two permanent space bases for Planetary Defense at the L1 and L3 points, and analyzed them mathematically. It was then shown that, placing missiles based at L1 and L3, would enable the missiles to deflect the trajectory of incoming asteroids by hitting them orthogonally to their impact trajectory toward the Earth, thus maximizing the deflection at best.

In this paper, however, we replace the missiles by the thrusters to be implanted into the asteroid's surface, thus avoiding the many political problems that putting missiles into space would have created with the public opinion.

Nomenclature

$M_{asteroid}$	=	mass of the asteroid in collision course with the Earth
$\rho_{asteroid}$	=	density of the asteroid
$D_{asteroid}$	=	diameter of the asteroid assumed to be spherical to simplify things
$V_{asteroid}$	=	asteroid speed with respect to the Earth when it enters the Earth's sphere of influence (V_∞)
R_{L1}	=	distance between the Earth and the Lagrangian point L1 of the Earth-Moon system assuming the Moon orbit to be circular ($R_{L1} \approx 323050$ km)
R_{L3}	=	distance between the Earth and the Lagrangian point L1 of the Earth-Moon system assuming the Moon orbit to be circular ($R_{L3} \approx 387135$ km)
$R_{Security}$	=	radius of Earth-centered sphere outside which we want to keep the asteroid at all times (20000 km)
$M_{missile}$	=	mass of the interceptor, i.e. the missile whose kinetic energy will deflect the asteroid by kinetic impact
$V_{ExtraBoost}$	=	interceptor's speed after the final acceleration against the asteroid, i.e. at the instant of deflection
V_1	=	notation for a speed used temporarily during the calculations, but discarded from the final formulae
V_2	=	notation for a speed used temporarily during the calculations, but discarded from the final formulae
E_P	=	kinetic energy of the interceptor at the instant of collision with the asteroid

I. Introduction

A rather innovative paper ([1]Maccone) was published in February 2002 by this author. It laid the mathematical foundations of a somehow new concept of Planetary Defense of the Earth against hazardous asteroids and comets. Further mathematical results along these lines were later published by the author in 2006 ([2]Maccone).
The guidelines of this new vision of the Planetary Defense are:

1) It is hard to deflect something that's coming right at you if you keep staying on the surface of the Earth. Thus, it is unreasonable to do any effective Planetary Defense from the surface of the Earth, and we must do it from space instead.
2) Where in space? The nearest two Lagrangian Points L1 and L3 of the Earth-Moon system are the two most obvious locations since they keep the same distance from the Earth at all times. We thus need to create missiles at bases there to be shot against hazardous asteroids. The two Earth-Moon Lagrangian Points L1 and L3 not only are quasi-stable points (i.e. any missile base there won't "fly away" into space) but most notably because the trajectory of all missiles shot from there will intercept the incoming asteroid at an angle of 90 degrees in all cases (confocal conics theorem). This "orthogonal interception" automatically insures the maximum sideway momentum transfer from the missile to the asteroid. In addition, a large steel basket open upon the missile head would help pushing the asteroid sideways, especially if the asteroid is small (<< 1 km in size). And such "small" rocks are the vast majority of the 364833 asteroids known on January 6th, 2007.
3) Not just that. A further advantage of the confocal conics theorem (used in ref. [1] to prove mathematically that the missile-asteroid collision would always occurs at an angle of 90 degrees) has one more consequence of exceptional practical importance that we call the "repeated deflection capability". Here is what this is: suppose that one missile-asteroid collision occurs, but the deflection in the asteroid's path is not large enough to bring it off its collision course with the Earth. Since the Earth always lies at the common focus of both the asteroid and missiles trajectories, the confocal conics theorem insures that all the elliptical trajectories of subsequent missiles are orthogonal to whatever hyperbolic trajectory the asteroid may have. This basic result means that we may shoot more the one missile in a sequence and have the asteroid deflected from one hyperbola to the next more eccentric one, and so on and so on for as many times as it may be needed until we finally push the asteroid off its collision course with the Earth. We like to call this result the "cumulative effect" of the repeated interception capability, and regard this "march of the dimes" of many smaller deflections totaling up into one, larger deflection as the key to save Humankind from the impact.

This introduction to the subject would not be complete without mentioning the several "popular descriptions" of the author's paper [1] that were published all over the world after February 2002. Most of these popular descriptions are nowadays downloadable from the Cambridge Conference Correspondence web site http://abob.libs.uga.edu/bobk/ccc/cc021402.html, but more similar, popular descriptions are possibly unknown to this author. The known-to-him popular summaries and comments were given by, respectively:

1) The popular science magazine "New Scientist" in an article titled "INCOMING! TO DEFLECT AN ASTEROID, CHOOSE YOUR SHOT CAREFULLY" by Eugenie Samuel. It is now available at the Cambridge Conference Correspondence site mentioned above. The author of this paper is grateful to Eugenie for her timely interest in his work and for her phone interview.
2) The senior science writer Robert Roy Britt wrote a popular description in the language of boxers! It is titled "Space-Based Missile Defense Needed to Thwart Asteroid Attacks" and can be found at the site http://www.space.com/scienceastronomy/solarsystem/deflection_asteroids_020214.html. The author of this paper is grateful to Robert Roy Britt for his neat article.
3) In German, a short description was later given in an article titled "Wie man Asteroiden abwehrt", available at the site http://www.pm-magazin.de/de/wissensnews/wn_id171.htm .

The author of this paper apologizes for not mentioning possible other "popular" web sites currently unknown to him.

It must be pointed out that this author's plan for Planetary Defense also has deep political and military implications, as it always happened for all new breakthroughs in the history of astronautics. To summarize them, we would like to use the same words that the British impact-expert Benny Peiser is reported (in Robert Roy Britt's article described above at point 4) to have said after having read the author's first paper [1]: "Peiser figures that a plan like Maccone's would have to be led by the U.S. Military, which already sees space as a necessary strategic outpost". The current response to terrorism, Peiser said, "has led the U.S. to significantly increase the budget for space-based defense paraphernalia which inadvertently enhances the prospects for advanced planetary defense technologies."

Finally, within the framework of the “New Trends in Astrodynamics” Conferences run by Prof. Ed Belbruno of Princeton University, a first account of this author’s results appeared in the 2004 paper entitled “Optimal Trajectories from the Earth-Moon L_1 and L_3 Points to Deflect Hazardous Asteroids and Comets”. Please see the web site: http://www.annalsnyas.org/cgi/content/abstract/1017/1/370 .

II. Asteroid Deflection Law in the Line/Circle Approximation

This section is the most important one in this paper. It contains the mathematical proof of the simple asteroid deflection law announced in the Abstract and given by Eq. (20) and Eq. (21) hereafter. The starting point is the remark that all Keplerian formulae proven by the author in ref. [3] may be greatly simplified when one realizes that:

1) The trajectory of the incoming asteroid or comet in between the distance of the Moon and the Earth is virtually a hyperbola very much coinciding with its own asymptote, namely a *straight line*. This straight line is supposed to hit the Earth by assumption (otherwise, we would have no interest to Planetary Defense!). In other words, this straight line is supposed to pass at a distance from the center of the Earth ranging between zero (a perfectly centered hit) and a certain minimum distance from the Earth that we call $R_{Security}$. This name is intended to remind that $R_{Security}$ is the *minimal* distance from the Earth center at which the incoming asteroid may pass without causing harm to humans. In the practice, $R_{Security}$ might be something like 20,000 km, so as to include also the lower atmosphere.

   Next, we regard the crash of the missile against the asteroid as non-elastic, and simply impose the conservation of linear momentum when this crash occurs. To this end, please consider Figure 1.

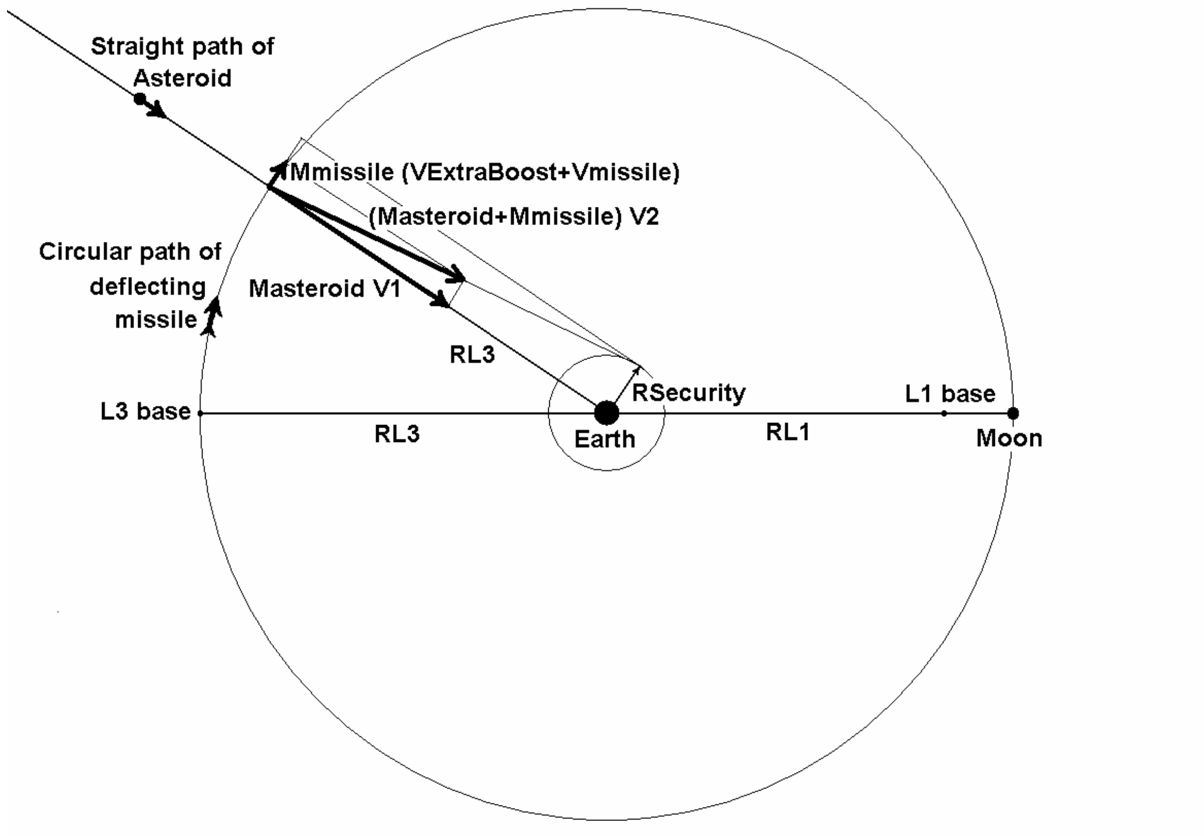

Figure 1. Derivation of the Asteroid Deflection Law in the Line/Circle Approximation. In this (fairly good) approximation to reality, the incoming asteroid is pointing straight to the Earth, at least from the distance of the Moon to the surface of the Earth. In other words, the incoming asteroid's hyperbola is virtually undistinguishable from its own asymptote, i.e. the path is a straight line. Then, the trajectories of the deflecting missiles launched from either L3 (as in this drawing) or L1 can only be circles centered at the Earth and passing through L3 or L1, respectively.

Figure 1 shows that there are two similar rectangular triangles having the same right angle at the Earth:

a) One rectangular triangle having smaller side $R_{Security}$ and longer side R_{L3} ;

b) Another rectangular triangle having smaller side $M_{missile}\left(V_{ExtraBoost}+V_{missile}\right)$ and longer side $V_1\ M_{asteroid}$.

Therefore, by rewriting this similarity in terms of modules of the vectors, we get the basic equation

$$\frac{R_{Security}}{R_{L3}}=\frac{M_{missile}\left(V_{ExtraBoost}+V_{missile}\right)}{V_1\ M_{asteroid}}. \tag{1}$$

Let us now consider this equation from the point of view of what one knows in it and what one doesn't. An immediate check shows that all variables in this equation are known from either the astronomical observations or from rocket technology, except one: the unknown asteroid mass $M_{asteroid}$ (although some very rough estimates about it could be possibly made in advance from the asteroid's size and spectra).

Therefore we decide to solve Eq. (1) for $M_{asteroid}$ and keep it ready for a later substitution into another equation:

$$M_{asteroid} = \frac{M_{missile}\left(V_{ExtraBoost} + V_{missile}\right) R_{L3}}{V_1\, R_{Security}} . \tag{2}$$

2) Since the trajectory of the incoming asteroid is now regarded to be a straight line, the only Earth-centered paths intercepting this straight line at 90 degrees are just ***circles*** centered at the Earth. And their radius is just either the L3 distance from Earth, R_{L3}, or, on the opposite side, the L1 distance from the Earth, R_{L1}. But the deflection still occurs at 90 degrees also in this simplified picture. The Pythagorean theorem thus applies among the modules of the intervening three linear momentum vectors, and one gets

$$\left(M_{asteroid} + M_{missile}\right)^2 V_2^2 = = \left(M_{asteroid}\right)^2 V_1^2 + \left(M_{missile}\right)^2 \left(V_{missile} + V_{ExtraBoost}\right)^2 . \tag{3}$$

Equations (2) and (3) are the only two equations needed in the line/circle approximation. Before we proceed, however, let us take a moment to think about the question: "which are the variables *in the power of humans to command* in order to achieve the minimal asteroid deflection?". The only plausible answer to this question appears to be: "In the two coupled equations (2) and (3) there is only one variable that we can select at will, and this is *the deflecting missile extra boost* $V_{ExtraBoost}$ **"**. In fact, all other variables are indeed determined by the law of gravitation, and there is nothing at all we can "command" there! It thus appears that the best thing to do is to solve Eq. (3) for $V_{ExtraBoost}$ so as to finally know from what remains, i.e. "from the law of gravitation", how much extra boost we must impart just prior to the missile crash on the asteroid in order to achieve the asteroid's minimal deflection. Solving Eq. (3) for $V_{ExtraBoost}$ yields

$$V_{ExtraBoost} + V_{missile} = = \frac{\sqrt{V_2^2\, M_{missile}^2 + 2\, V_2^2\, M_{missile}\, M_{asteroid} + \left(V_2^2 - V_1^2\right) M_{asteroid}^2}}{M_{missile}} . \tag{4}$$

Let us now take a moment to check which numbers are "big" and which ones are "small" in the last equation. Figure 1 shows that the minimal deflection angle (at the Moon distance) is very small, something like 1.5 degrees for $R_{Security} = 10{,}000$ km. Therefore the hypotenuse in the rectangular triangle of the three linear momenta is just *slightly* longer than the longer triangle side. In other words, numerically speaking, we can *approximate* Eq. (4) by safely assuming that

$$V_2 - V_1 \approx 0 \quad \text{that is} \quad V_2 \approx V_1 . \tag{5}$$

By doing so, we thus get rid of the variable V_2. And the approximated version of Eq. (4) reduces to

$$V_{ExtraBoost} = \frac{V_1 \sqrt{M_{missile} + 2\, M_{asteroid}}}{\sqrt{M_{missile}}} - V_{missile} . \tag{6}$$

In this equation, all quantities are known except for the asteroid's mass $M_{asteroid}$. But the asteroid's mass is given by Eq. (2). Replacing thus Eq. (2) into Eq. (6) one finds the new equation

$$V_{ExtraBoost} + V_{missile} = \frac{V_1 \sqrt{M_{missile} + 2\, \dfrac{M_{missile}\left(V_{ExtraBoost} + V_{missile}\right) R_{L3}}{V_1\, R_{Security}}}}{\sqrt{M_{missile}}} \tag{7}$$

Rearranging, this becomes

$$V_{ExtraBoost} + V_{missile} = \frac{\sqrt{V_1}\,\sqrt{V_1\,R_{Security} + 2\left(V_{ExtraBoost} + V_{missile}\right) R_{L3}}}{\sqrt{R_{Security}}}\,. \tag{8}$$

A glance to Eq. (8) reveals that it actually leads to an algebraic second-degree equation in the unknown variable $V_{ExtraBoost}$. By rearranging, squaring to get rid of radicals, and expanding, one gets:

$$\left(V_{ExtraBoost} + V_{missile}\right)^2 = \frac{V_1^2\,R_{Security} + 2\,V_1\left(V_{ExtraBoost} + V_{missile}\right) R_{L3}}{R_{Security}}\,. \tag{9}$$

By expanding again and factoring, the algebraic second-degree equation in $V_{ExtraBoost}$ is thus obtained

$$R_{Security}\,V_{ExtraBoost}^2 + 2\left(V_{missile}\,R_{Security} - V_1\,R_{L3}\right) V_{ExtraBoost} + \left(V_{missile}^2 - V_1^2\right) R_{Security} - 2\,V_1\,V_{missile}\,R_{L3} = 0\,. \tag{10}$$

This yields the two roots

$$\begin{cases} V_{ExtraBoost} + V_{missile} = \dfrac{V_1\left(R_{L3} + \sqrt{R_{Security}^2 + R_{L3}^2}\right)}{R_{Security}} \\ V_{ExtraBoost} + V_{missile} = \dfrac{V_1\left(R_{L3} - \sqrt{R_{Security}^2 + R_{L3}^2}\right)}{R_{Security}}. \end{cases} \tag{11}$$

The second root in Eq. (11) is unphysical, since it would yield and negative right-hand side. Therefore we must retain the first root only, and so we find

$$V_{ExtraBoost} = \frac{V_1\left(\sqrt{R_{Security}^2 + R_{L3}^2} + R_{L3}\right)}{R_{Security}} - V_{missile}\,. \tag{12}$$

This is our first important result. It allows us to achieve the minimal deflection for any asteroid having an incoming velocity V_1. A table of known incoming velocities V_1 for actual hazardous asteroids is found at the NASA-JPL web site http://neo.jpl.nasa.gov/ca/ . The table shows that

$$4 \text{ km/sec} < V_1 < 40 \text{ km/sec}. \tag{13}$$

On the other hand, the variable $V_{missile}$ appearing in Eq. (12) simply is the missile's circular velocity around the Earth at the L3 distance and is thus given by

$$V_{missile} = \sqrt{\frac{G\,M_{Earth}}{R_{L3}}} \approx 1.022 \text{ km/sec}, \tag{14}$$

a rather insignificant value (= up to 40 times smaller) with respect to the incoming asteroid speed, V_1.

Let us now complete the solution of the two coupled equations (2) and (3) by replacing Eq. (12) into Eq. (2). Solving then the resulting equation for $M_{missile}$ one finds

$$M_{missile} = \frac{M_{asteroid}\ R^2_{Security}}{R_{L3}\left(\sqrt{R^2_{Security} + R^2_{L3}} + R_{L3}\right)}. \quad (15)$$

Although $M_{asteroid}$ is unknown, this equation is important result inasmuch as it actually represents the linear function $M_{missile} = M_{missile}(M_{asteroid})$. In words, Eq. (15) tells us "how big the missile must be in order to achieve the minimal deflection for any given asteroid". We are thus led to consider the question "how big is the asteroid?".

This question may be better understood in terms of asteroid size, rather than in terms of asteroid mass. And to shift from the asteroid mass to its size we are going to make one more simplifying assumption (although clearly a debatable one): that the asteroid's shape is *spherical*. For a sphere, one then obviously has

$$\rho_{asteroid} = \frac{M_{asteroid}}{\frac{4}{3}\pi\ R^3_{asteroid}}. \quad (16)$$

For a sphere again, the size of the asteroid is its diameter, related to the asteroid's radius by $D_{asteroid} = 2\ R_{asteroid}$. By solving the last two coupled equations for $M_{asteroid}$, one gets

$$M_{asteroid} = \frac{\pi\ \rho_{asteroid}\ D^3_{asteroid}}{6}. \quad (17)$$

Finally, replacing Eq. (17) into Eq. (15), one finds

$$M_{missile}(D_{asteroid},\quad \rho_{asteroid}) = \frac{\pi\ \rho_{asteroid}\ D^3_{asteroid}\ R^2_{Security}}{6\ R_{L3}\left(\sqrt{R^2_{Security} + R^2_{L3}} + R_{L3}\right)}. \quad (18)$$

This is the second, basic formula (next to Eq. (12)) in the Line/Circle approximation of the asteroid deflection theory. It tells us "how big" the deflecting missile must be in order to achieve the minimal deflection of any "that big" asteroid.

One final step only is requested in order to find the third, really fundamental result of this paper, that we call the "deflection law". This step can only be the merging of the two important results obtained so far, namely the merging of Eq. (12) and Eq. (18). By eliminating the parenthesis where the radical is, one thus obtains

$$V_{ExtraBoost} = \frac{\pi V_1\ \rho_{asteroid}\ D^3_{asteroid}\ R_{Security}}{6\ M_{missile}\ R_{L3}} - V_{missile}. \quad (19)$$

This is our Asteroid Deflection Law: a key equation to achieve the Planetary Defense within the Earth-Moon System in Space, rather than from the Earth!

Yet, this equation may still be simplified a little. In fact, the missile velocity, $V_{missile}$, given by Eq. (14), simply is the orbital speed of the missile around the Earth at the distance of L3, and so it is very small (~ 1 km/sec) when compared to the much higher speed $V_{ExtraBoost}$ requested to achieve the asteroid deflection (~ tens or hundreds of km/sec, or more). Therefore, we may further simplify the Deflection Law simply by forgetting the $V_{missile}$ in it. Thus, *the ASTEROID DEFLECTION LAW for missiles shot from L3 reads*

$$V_{ExtraBoost} \approx \frac{\pi V_{asteroid}\ \rho_{asteroid}\ D^3_{asteroid}\ R_{Security}}{6\ M_{missile}\ R_{L3}}. \quad (20)$$

Similarly, *the ASTEROID DEFLECTION LAW for missiles shot from L1 reads*

$$V_{ExtraBoost} \approx \frac{\pi\, V_{asteroid}\ \rho_{asteroid}\ D^3_{asteroid}\ R_{Security}}{6\, M_{missile}\, R_{L1}} . \tag{21}$$

These two equations are the key results of our Space Planetary Defense System, still within the Earth-Moon gravitational system.

As a corollary, it is easy to find how the relative errors affecting the variables in our Deflection Laws are related to each other. Just take the natural log of both sides of either Eq. (20) or Eq. (21). Then differentiate both sides, by obviously setting to zero the differential of any variable not changing in the analysis of the deflection problem. Then, a few trivial steps lead to the formula

$$\frac{\Delta V_{ExtraBoost}}{V_{ExtraBoost}} \approx \frac{\Delta V_{asteroid}}{V_{asteroid}} + \frac{\Delta\rho_{asteroid}}{\rho_{asteroid}} + 3\cdot\frac{\Delta D_{asteroid}}{D_{asteroid}} + \frac{\Delta R_{Security}}{R_{Security}} - \frac{\Delta M_{missile}}{M_{missile}} \tag{22}$$

This formula, too, will be pivotal to design an efficient Planetary Defense System by missiles shot from L1 and L3.

III. Avoiding the Asteroid's Fragmentation during the Kinetic Impact

In this section we study how to avoid the asteroid's fragmentation during the kinetic impact by the interceptor. This problem was first faced by Thomas J. Ahrens of Caltech and Alan W. Harris of JPL back in 1994 ([3]Ahrens, T. J, and Harris, A. W). Ten years later, their equations were embodied in the NASA/TP – 2004-213089 by Robert B. Adams et al. ([4]Adams, R. B. et al.) and also coded in the form of an Excel spreadsheet. Finally, in January 2007 this author converted that Excel spreadsheet in MathCad form and merged the equations given by Ahrens and Harris with the above Eqs. (20) and (21). The result is a new equation proving an upper bound on the mass (i.e. on the size and density) of the asteroids that can be deflected from the two proposed permanent space bases at L1 and L3. That such an upper bound had to exist was intuitively obvious, but our new equation yields mathematical light to the Planetary Defense System proposed in this paper.

Let us start by pointing out that, in the NASA/TP of 2004, the condition for fragmentation or non-fragmentation of the asteroid is that the ratio between the kinetic energy of the interceptor and the asteroid's mass is larger or smaller than 0.5 joules/gram, respectively. In other words, denoting the interceptor's kinetic energy by E_P, the *condition for non-fragmentation* (also called *condition for craterization*) reads

$$\frac{\dfrac{E_P}{M_{asteroid}}}{\dfrac{joule}{gram}} \le 0.5 . \tag{23}$$

Now, in view of our deflection law, Eq. (20), the kinetic energy of the interceptor (or "missile" in the language of the previous section in this paper) is clearly given by

$$E_P = \frac{1}{2} M_{missile} V^2_{ExtraBoost} = \frac{\pi^2\ V^2_{asteroidt}\ \rho^2_{asteroid}\ D^6_{asteroid}\ R^2_{Security}}{72\, M_{missile}\, R^2_{L3}} . \tag{24}$$

On the other hand, the mass of the asteroid is given by Eq. (17). Thus, dividing Eq. (24) by Eq. (17), one gets the new expression for the numerator of the non-fragmentation condition (23)

$$\frac{E_P}{M_{asteroid}} = \frac{\pi\, V^2_{asteroid}\ \rho_{asteroid}\ D^3_{asteroid}\ R^2_{Security}}{12\, M_{missile}\, R^2_{L3}} . \tag{25}$$

Finally, replacing the 0.5 in the right-hand side of Eq. (23) by ½, and replacing Eq. (25) into the inequality of Eq. (23), one gets the expression of the *no-fragmentation condition*

$$\frac{\pi V_{asteroid}^2 \ \rho_{asteroid} \ D_{asteroid}^3 \ R_{Security}^2}{6 \, M_{missile} \, R_{L3}^2} \leq 1 \cdot \frac{joule}{gram} . \tag{26}$$

In this formula we must now point out which factors in the left-hand side are under the control of humans and which factors are not so. Strictly speaking, only $M_{missile}$ is under the control of humans. However, R_{L3} and $R_{Security}$ may be regarded as "unchangeable" quantities also, i.e. not depending on which asteroid comes along. Therefore, we may rewrite the inequality in Eq. (26) as

$$V_{asteroid}^2 \ \rho_{asteroid} \ D_{asteroid}^3 \leq \frac{6}{\pi} \cdot M_{missile} \cdot \left(\frac{R_{L3}}{R_{Security}} \right)^2 \cdot \frac{joule}{gram} . \tag{27}$$

Now, the density of all asteroids almost always lies in between 1 gram/cm^3 and 4 gram/cm^3. Therefore we may regard $\rho_{asteroid}$ in Eq. (27) as a "parameter". We may then solve Eq. (27) for the function $V_{asteroid}(D_{asteroid})$ and plot such a function for values of the parameter $\rho_{asteroid}$ ranging in between 1 and 4 gram/cm^3. The result is the inequality

$$V_{asteroid} \ \leq \frac{1}{\sqrt{D_{asteroid}^3}} \cdot \frac{1}{\sqrt{\rho_{asteroid}}} \cdot \sqrt{\frac{6}{\pi} \cdot M_{missile} \cdot \left(\frac{R_{L3}}{R_{Security}} \right)^2 \cdot \frac{joule}{gram}} . \tag{28}$$

The following four numerical plots show each the four $V_{asteroid}(D_{asteroid})$ curves obtained by replacing the inequality sign by an equality sign into Eq. (28) and letting $\rho_{asteroid}$ range from 4 gram/cm^3 (curve closest to the origin of the axes) to 1 gram/cm^3 ("outer" curve).

The meaning of these four plots is as follows:

Figure 2 shows the asteroid diameter on the horizontal axis (in meters) and, on the vertical axis, the maximum asteroid speed at which a no-fragmentation kinetic deflection could be achieved by a missile having a mass of 1500 kg. The results are disappointing: only asteroids arriving with a speed (with respect to the Earth) of less than 0.5 meters per second (and not kilometers per second!) could be deflected kinetically by such a missile by at least the angle of 2.96 degrees subtended by our $R_{Security}$ of 20000 km at the distance of the L3 point (387135 km)! The conclusion is obvious: *no kinetic deflection (without fragmentation) can be achieved from L3 because the incoming asteroids have speeds about three orders of magnitude higher than the no-fragmentation limit!*

In other words still, only nuclear deflection would work from the distance of L3, i.e. the Planetary Defense base at L3 (and the one at L1) should be endowed with nuclear warheads to achieve deflection. Not a nice perspective at all, in view of the political problems that such a need would cause! Then, only one alternative solution seems to exist: sticking thrusters into asteroids to deflect them at will. But this must be done outside the sphere of influence of the Earth! Please note that asteroid (99942) Apophis, estimated to be 320 meters in diameter with an incoming speed of 12 km/sec at the (theoretical) impact with the Earth, falls into this category of non-kinetically-deflectable asteroids for sure! The threat of Apophis must be faced away from the Earth, in the open space and with a lead time of decades or more. It might then be possible to deflect it even with a 1500 kg kinetic interceptor shot at a suitable distance from Apophis so as not to fragment it but also such as to optimize the kinetic energy available.

Figure 3: the same as Figure 2 for asteroids ranging in between 3 and 100 meters in diameter.

Figure 4: the same as Figure 3 for asteroids ranging in between 3 and 50 meters in diameter,

Figure 5: the same for the smallest rocks up to 10 meters. Not even for them would the kinetic deflection work!

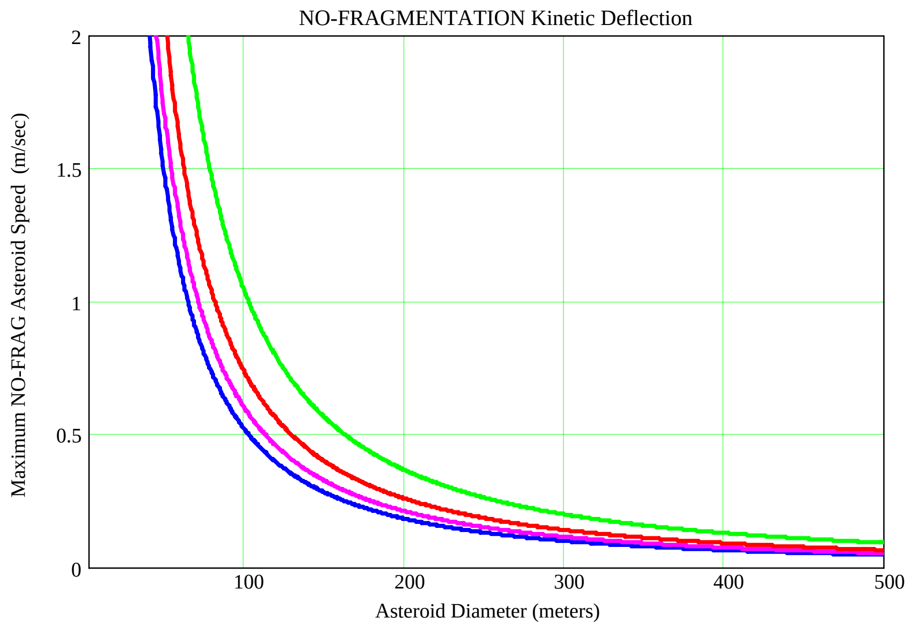

Figure 2.

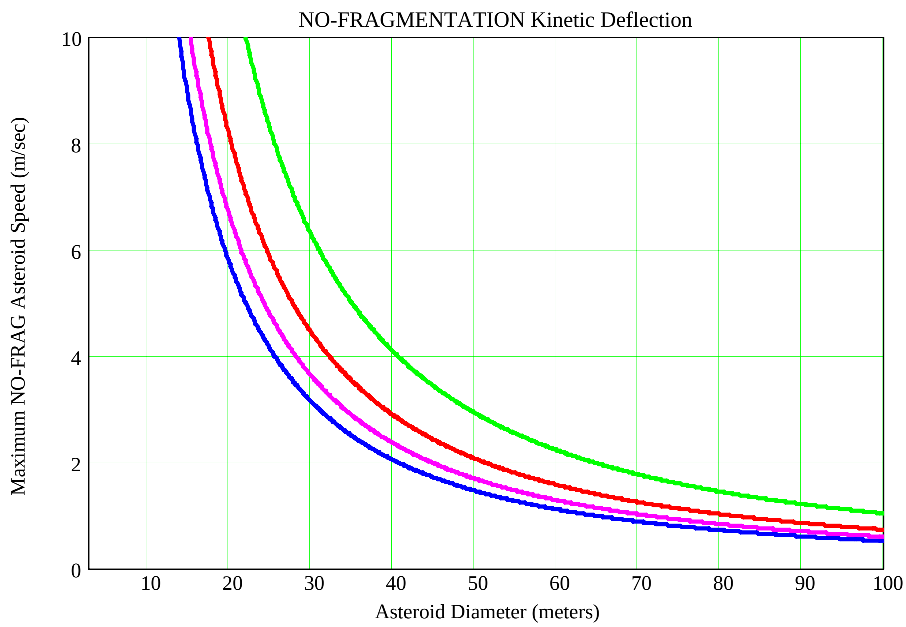

Figure 3.

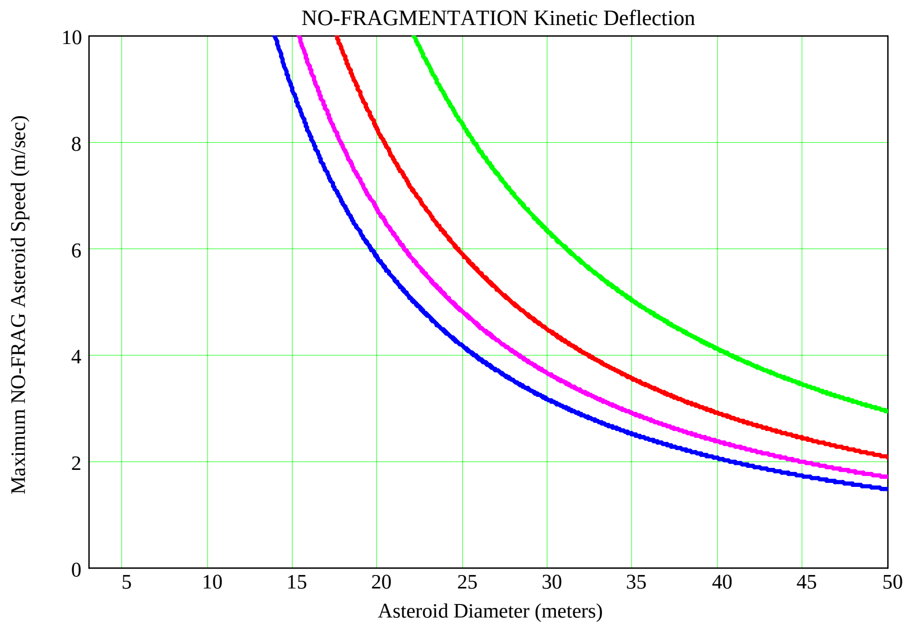

Figure 4.

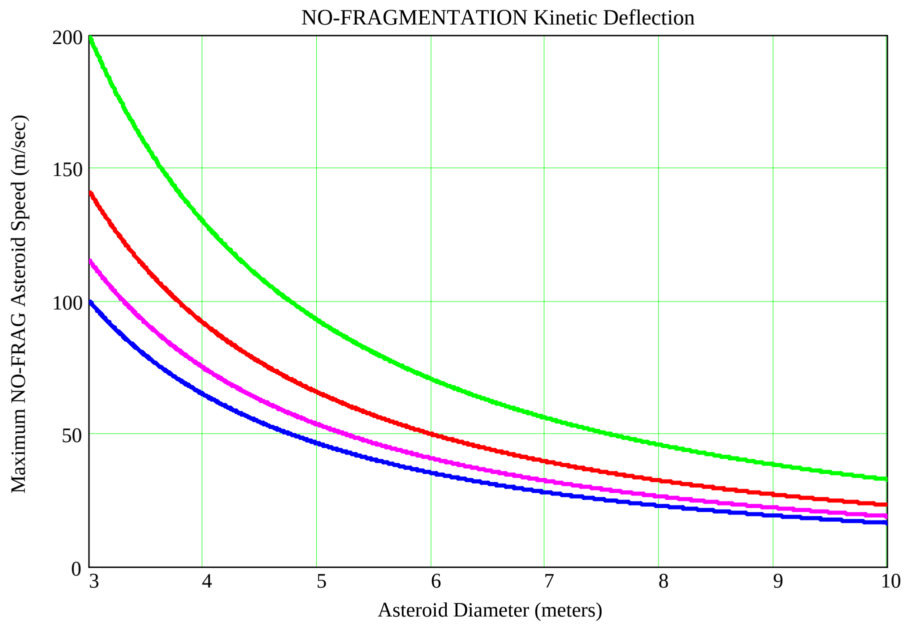

Figure 5.

IV. Conclusion: Sticking Thrusters into Asteroid in the Open Space

The conclusion is that it seems *impossible* to safely stick thrusters into an asteroid by shooting them from (future) Space Bases at the Lagrangian points L1 and L3 of the Earth-Moon system quite simply because all asteroids arrive with a much too high speed.

In other words, shooting missiles at incoming asteroids from L1 and L3 would cause their fragmentation into pieces, rather than their deflection by the requested amount to avoid collision with the Earth.

It is certainly possible, however, to stick thrusters into asteroids into the open space (i.e. outside the gravitational sphere of influence of the Earth) by

1) putting a spacecraft on the same heliocentric orbit as the asteroid, so that both keep staying at a convenient and constant distance from each other;
2) shooting a (say 1500 kg) "bullet" from the spacecraft so that its kinetic energy is smaller than the fragmentation limit for the asteroid, and only causes craterization and deflection.
3) shoot more such bullets (without asteroid fragmentation) until the asteroid has been successfully deflected by the requested amount.

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