

Covariance Analysis of Near Earth Objects (NEOs)

V. Nuth¹, P. V. Sheth², J. W. Cook³, P. D. Martzen⁴, J. E. Gidney, Jr.⁵ and H. F. Fliegel⁶
The Aerospace Corporation, El Segundo, CA, 90245

The ability to determine and predict NEO orbits accurately is essential to deterring a collision threat of these potentially hazardous objects (PHOs). The Aerospace Corporation's orbit determination and analysis tool, TRACE, can be used to evaluate the effectiveness of ground and space-based systems to detect and characterize NEOs -an asteroid or comet that at some point in its orbit comes close to Earth. This study will incorporate observation systems from Earth and deep space for the detection of NEOs 140 meters in diameter and larger and is performed to determine how well the trajectory of a NEO can be estimated. Throughout the study, tracking of each NEO is assumed to be from a deep space platform (i.e., near Venus orbit) as well as from the Earth using angle and range measurements. Orbit Determination (OD) accuracies are generated via covariance analyses using TRACE. A variety of tracking and visibility constraint scenarios are evaluated, and include the sun exclusion angle and the apparent magnitude of the PHO from the tracker. The results presented in this paper are preliminary and based on a single asteroid, Apophis. Future study will include all of the objects identified in the Jet Propulsion Laboratory NEO database.

I. Introduction

PREDICTING a NEO's position and velocity with high precision is critical to deflect these objects effectively from a future Earth collision. A NEO may be tracked using various measurement types from a number of tracking locations. Relevant tracking locations must be evaluated based on geometric factors, measurement quality, mission feasibility, implementation cost, and other factors to determine their advantages. For this study, the evaluation criteria are measurement quality and geometric factors only. To assess a set of tracking scenarios, the degree of precision obtained for each scenario should be based on a population of NEOs. Since this study is still in its development phase, the analysis presented in this paper is focused on evaluating the tracking scenarios for a single NEO only.

The asteroid Apophis is the NEO used for this paper's analysis. In the recent past, Apophis posed a potential threat for an Earth collision and is still of major concern today. Tracking locations will include an Earth-based ground station and a satellite tracker near the Venus orbit. A real-time covariance analysis performed by The Aerospace Corporation's trajectory analysis and orbit determination program, TRACE (Ref. 1), is used to determine the accuracy to which Apophis' position can be determined from the various tracking scenarios. A total of four scenarios are evaluated for this study and include combinations among the two tracking locations and different measurement types. The epoch date for all cases is midnight June 14, 2006.

II. TRACE and Covariance Analysis

In operation over 40 years, TRACE assists in the design and analysis of complex problems related to orbital dynamics, observation modeling, and estimation theory. TRACE supports analysis of numerous orbit related problems; namely, ephemeris propagation, orbit determination, measurement generation, and error or covariance analysis. TRACE continues to evolve in order to support better force modeling, additional measurement models,

¹ Sr. Member of the Technical Staff, Navigation and Geopositioning Systems Department, M4/962.

² Member of the Technical Staff, Navigation and Geopositioning Systems Department, M4/962.

³ Ret. Member of the Technical Staff, Navigation and Geopositioning Systems Department, M4/962.

⁴ Sr. Engineering Specialist, Navigation and Geopositioning Systems Department, M4/962.

⁵ Director, Navigation and Geopositioning Systems Department, M4/944.

⁶ Ret. Member of the Technical Staff, Navigation Division, M5/689.

and enhanced estimation algorithms. TRACE has been used extensively for Earth orbiting satellites and is regarded as a high precision analysis tool. Here, we use TRACE for the first time to analyze orbital accuracy for a NEO-type problem.

The two primary functions we use in this study are the observation generation and covariance analysis. The observation generation simulates the measurements of Apophis from a specified ground site as well as from near Venus orbit. The measurement types generated are “range” (meant to simulate a radar system) and “angles” (meant to simulate an optical system). TRACE can model the effects of random noise and bias error in the observations. The covariance analysis function in TRACE can be viewed as a design tool. In short, it provides the capability of analyzing the effect of various level of tracking error uncertainties and the *a priori* state uncertainties associated with a specified orbit.

The accuracy of an estimation system is dependent on the accuracy with which the dynamics and the measurements used to observe the system can be modeled. In order to achieve a level of accuracy, one must begin by identifying the important error sources and their statistical description. The covariance analysis serves to characterize the sensitivity of the estimation accuracy to these error sources. In the context of this study, the covariance analysis is used to gauge the potential accuracy of an OD process for different tracking scenarios (i.e., tracker locations, data types, and associated accuracies).

The formulation of the covariance matrix in TRACE follows the standard weighted least squares (WLS) problem (Ref. 2); that is, the inverse of the normal matrix is the covariance matrix of interest. The TRACE covariance matrix allows the partition of two different classes of parameters, the estimated parameters, P, and the un-estimated (or consider) parameters, Q. The Q parameters are generally associated with errors due to imperfect force models (e.g., gravity field, drag, and solar radiation pressure) and measurement bias uncertainty. The primary limitation of the covariance analysis is the ability to quantify the error in the un-estimated error sources. However, the resulting prediction of the orbit error is generally more realistic when using available estimates of these error values rather than assuming that they are zero. It should be noted that this study uses the real-time mode of the TRACE covariance analysis, meaning that each data point is processed as it is received as opposed to processing a batch of data for the entire OD time span.

III. Near Earth Objects

To date, approximately 4,000 NEOs have been discovered in our solar system (Ref. 3). Most of these NEOs consist of asteroids, also known as Near Earth Asteroids (NEAs). The NEA Apophis will be used as an example object for the analysis presented in this paper. The asteroid orbit’s semi-major axis lies less than 1.0 AU from the Sun and it takes Apophis approximately 323 days to make a full revolution around the Sun. The asteroid itself is about 415 meters across and has a slightly elliptical orbit with an eccentricity of 0.191 and an inclination of 3.33 degrees (Ref. 4). Figure 1 shows the orbit of Apophis and the Earth relative to the sun starting from the epoch date. The initial conditions used for Apophis in the covariance analysis are captured in both the Earth-Centered Inertial and Heliocentric Inertial frames in Table 1.

Table 1 : Apophis Initial Conditions

Parameter	(HCI)	(ECI)
Epoch	6/14/06 00:00:00 GMT	6/14/06 00:00:00 GMT
X (km)	43369776.008207	62985596.733
Y (km)	-106972225.014520	37424371.53
Z (km)	6713079.004814	23547949.649
Xdot (km/s)	32.233020518	3.179075
Ydot(km/s)	17.148195937	19.414682
Zdot (km/s)	-0.131705415	8.273693

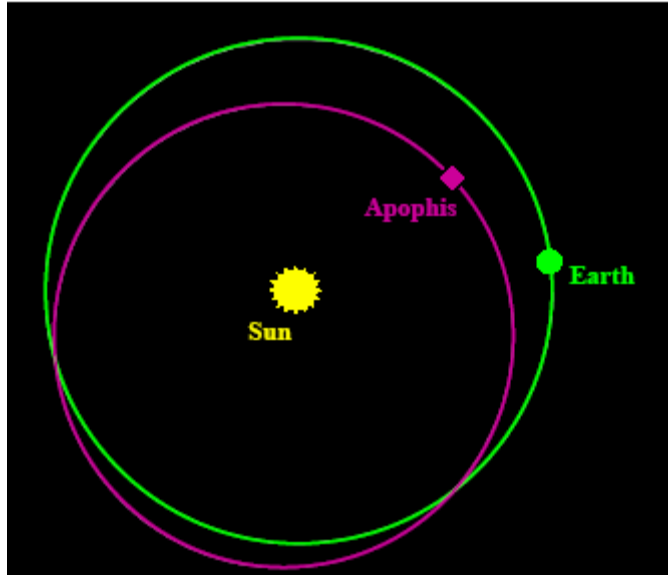


Figure 1 : Apophis' orbit about the Sun. Diamond represents position on June 14, 2006.

IV. Covariance Assumptions

As mentioned earlier, a total of four cases are evaluated for this study and include combinations of the two tracking locations and different measurement types. Each tracking scenario does not always have visibility of Apophis. For this reason, the measurements from the trackers are filtered based on apparent magnitude and solar elongation angle restrictions before being used for the covariance analysis. These values are based mainly on the geometry between the tracker, Apophis, and the Sun. Also, various assumptions must be made to evaluate the desired tracking scenarios. More details on these assumptions are addressed in the sections below.

The position and velocity of an asteroid cannot be known exactly due to measurement sensor errors and various other uncertainties. For this reason, *a priori* uncertainties for the initial position and velocity of Apophis were incorporated into the TRACE covariance analysis. The orbit state uncertainties used in the analysis are shown in Table 2.

The *a priori* state uncertainties are implemented for both numerical stability and effective tracking scenario evaluation. Without *a priori* uncertainties, the covariance analysis would be subject to numerical instability due to near zero values in each of the elements in the normal matrix, since it must be inverted to ascertain the covariance matrix. The values for the *a priori* uncertainties are chosen conservatively so that they would not significantly impact the results by under-weighting the measurements. The values unique to each tracking scenario are uncertainties in the force models, tracker sensor noises and biases, and tracker geometry. Having the *a priori* state uncertainties set to a less conservative value would effectively weaken the apparent uniqueness of uncertainties in each tracking scenario. The purpose of this paper's analysis is to evaluate how well each tracking case reduces the initial uncertainties in Apophis' position.

Table 2 : Apophis *a priori* State Uncertainties

Parameter	Uncertainty (1-sigma)
x	1000 km
y	1000 km
z	1000 km
x $\dot{}$	1 km/s
y $\dot{}$	1 km/s
z $\dot{}$	1 km/s

V. Tracking Locations

Two tracking locations are chosen in this analysis. The first is an Earth-based station located in Santiago, Chile. The second is a satellite tracker that is near the Venus orbit. It is assumed that the measurement frequency is every 10 days for a time span of 1000 days starting from the epoch date. These measurements are filtered based on visibility constraints before the covariance analysis.

A. Earth Station

The Earth-based station used for this analysis is located on the grounds of the Santiago tracking station in Santiago, Chile. Operated by the Center for Space Studies (CEE, Centro de Estudios Espaciales), this facility hosts satellite tracking equipment and conducts research in disciplines connected with earth sciences, earth resources, and the environment. TRACE has the ability to model ground stations and specify the ground station parameters as well as sensor parameters. The position and altitude of the ground station as input into TRACE are shown in Table 3 (Figure 2).

Table 3 : Position and Altitude of Santiago, Chile Ground Station

Earth Station Location	North Latitude (deg)	East Longitude (deg)	Station height (km) (above mean sea level)
Santiago, Chile	-33.00	289.00	0.73



Figure 2 : Location of Santiago, Chile

The Earth-based station is capable of collecting optical measurements, radar measurements, or both. The optical measurements are used to obtain right ascension and declination angles for the asteroid whereas radar is used to obtain range measurements. The measurement noise and bias uncertainties (Ref. 5) for the sensors are shown in Table 4 below. These bias uncertainties are treated as consider parameters in the covariance runs. Figure 1 illustrates the position of Apophis and the Earth relative to the Sun on the epoch date (June 14, 2006), when looking down upon the ecliptic plane.

Table 4 : Noise and Bias Uncertainties for Measurements from Earth Ground Station

Earth Station : Santiago Chile		
Measurement Type	Noise (1-sigma)	Bias Uncertainty (1-sigma)
Right Ascension	0.21 arc sec	0.105 arc sec
Declination	0.21 arc sec	0.105 arc sec
Range	100 m	100 m

B. Near-Venus Orbit Tracker

The near-Venus orbit tracker is located approximately 0.723 AU from the Sun. Analysis is performed using this particular tracker in order to assess any potential benefits or disadvantages of having a tracker in deep space. Figure 3 shows the tracker's position on the epoch date with respect to the Sun and the Earth. The initial conditions used for the near-Venus orbit tracker as input into TRACE are shown in Table 5.

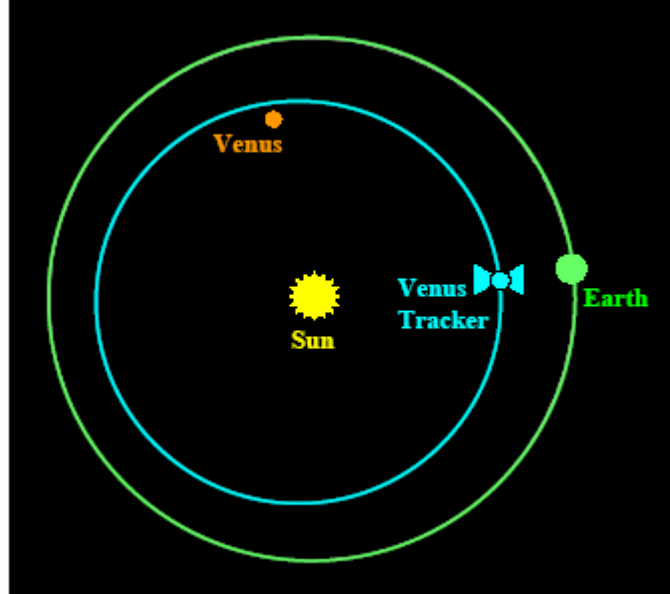


Figure 3 : Near-Venus Orbit Tracker with respect to Venus, Sun, and Earth

Table 5 : Initial Conditions for Near-Venus Orbit Tracker

Parameter	(HCI)	(ECI)
Epoch	6/14/06 00:00:00 GMT	6/14/06 00:00:00 GMT
x (km)	-14445821.0	5170000.0
y (km)	-109559401.0	37736000.0
z (km)	-37819.0	1632500.0
\dot{x} (km/s)	35.0	5.945651
\dot{y} (km/s)	-4.627198.0	0.13909
\dot{z} (km/s)	-1.897114.0	-2.007425

The near-Venus orbit satellite tracker is assumed to be capable of optical measurements, yielding right ascension and declination values for the NEO. The measurement noise and bias uncertainties (Ref. 5) for the sensors are shown in Table 6 below and are the same values as those for the Earth station. These biases are also treated as consider parameters in the covariance runs.

Table 6 : Noise and Bias Uncertainties for Measurements from Near-Venus Orbit Tracker

Near-Venus Orbit Tracker		
Measurement Type	Noise (1-sigma)	Bias Uncertainty (1-sigma)
Right Ascension	0.21 arc sec	0.105 arc sec
Declination	0.21 arc sec	0.105 arc sec

The Near-Venus Orbit Tracker position and velocity uncertainties are also included in the covariance analysis as consider parameters. Their values are 0.001 km and 1.0e-9 km/s, respectively.

VI. Force Model Assumptions

A. Reference Frame, Central Body, and Planetary Perturbations

Due to the deep space nature of the various tracking cases in this study, the covariance analysis and trajectory integration for the various objects would ideally be performed in the Heliocentric-Inertial (HCI) frame with the Sun acting as the central body. Using current capabilities in TRACE, the covariance analysis is performed in the Earth-Centered inertial (ECI) frame with the Earth acting as the central body. The Sun and all of the planets are treated as perturbations in the force model. Efforts are currently underway to incorporate a method in TRACE to allow estimated and consider parameter uncertainties to be input for the location and mass of the Sun, Moon, and planets. Since the primary purpose of this study is to evaluate the ability of tracking scenarios to reduce uncertainty in position, it is assumed that treating the Earth as a central body and treating the Sun, Moon, and planets as planetary perturbations suffices for comparing various tracking scenarios.

B. Earth Gravitational Constant

The Earth gravitational constant, a physical quantity that is inherently difficult to measure, is also included as an estimated parameter in the TRACE analysis. The value of GM and its associated uncertainty are shown in Table 7. These values represent standard values obtained from the International Earth Rotation and Reference System Service (IERS) Technical Note 21(Ref. 6).

Table 7 : Gravitation Constant Values

Parameter	Value
GM (km ³ /s ²)	398600.4418
GM (km ³ /s ²) Uncertainty	0.83E-3

C. Solar Radiation Pressure Constant

Solar radiation pressure is a force that invokes perturbations on objects in the solar system. Since this value is not absolutely known, it is included as an estimated parameter for this analysis. The *a priori* uncertainty for the solar radiation pressure is assumed to be within 20 percent of the solar radiation pressure value. Their quantities are shown in Table 8 below.

Table 8 : Solar Radiation Pressure Constant Values

Parameter	Value
Solar Radiation Pressure (ft ² /lbm)	0.8E-10
Solar Radiation Pressure <i>a priori</i> Uncertainty (ft ² /lbm)	0.16E-10

VII. Masking

To assess the advantages of the various tracking scenarios based on their geometrical location, visibility constraints are applied to each tracking scenario case. The constraints include restrictions in asteroid viewing due to the apparent magnitude of the asteroid and the size of the solar elongation angle between the Sun, tracker, and asteroid.

A. Apparent Magnitude

The apparent magnitude of an observed object represents its apparent brightness as seen by a specified observation point. This value is based not only on the distance between the two points but also by the size and characteristics of the observed object itself. The higher the apparent magnitude value, the fainter the object appears from the observation point. In order to account for this visibility constraint, measurements taken at a time when the object's apparent magnitude is above a maximum threshold value of 25 are not used. Figure 4 illustrates the affect of distance on apparent magnitude of an asteroid from an arbitrary tracking location.

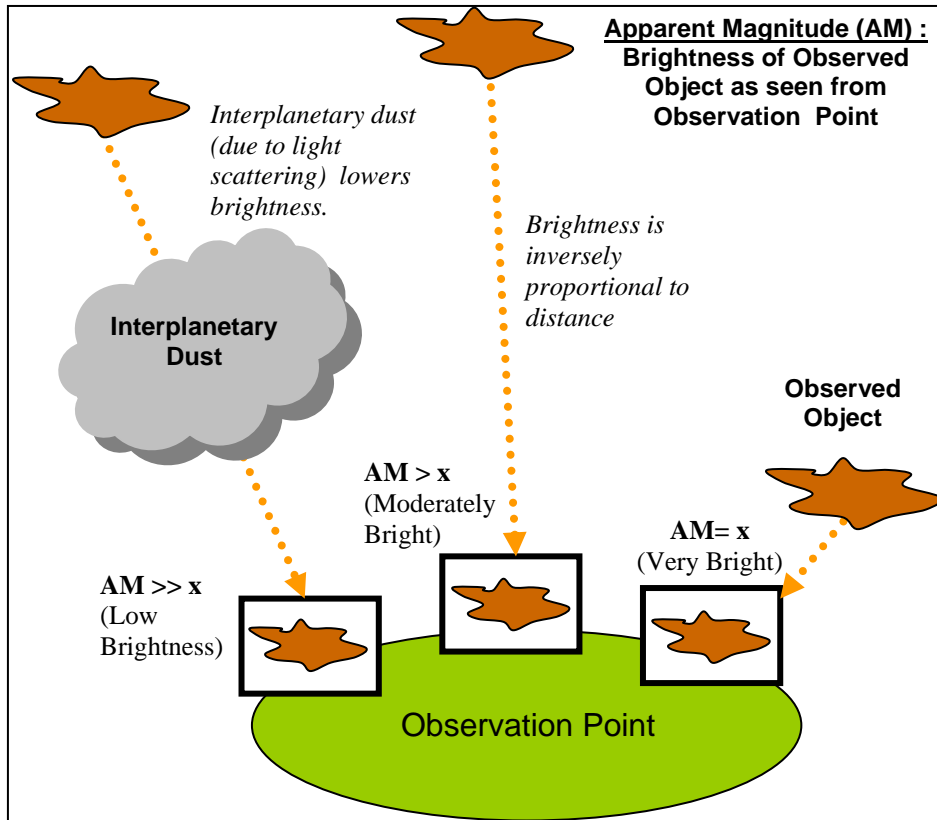


Figure 4 : Illustration of Distance's affect on Apparent Magnitude

B. Solar Elongation Angle

The ability to acquire measurements from a tracking location may be impeded by the Sun's brightness and its ability to over-saturate the camera or recording device taking the measurement. Therefore, constraints are placed on each tracking scenario to account for the solar elongation angle. If the solar elongation angle reaches a set minimum value of 50 degrees, those measurements are rendered invalid and are not used for the covariance analysis. Figure 5 illustrates the definition of the solar elongation angle.

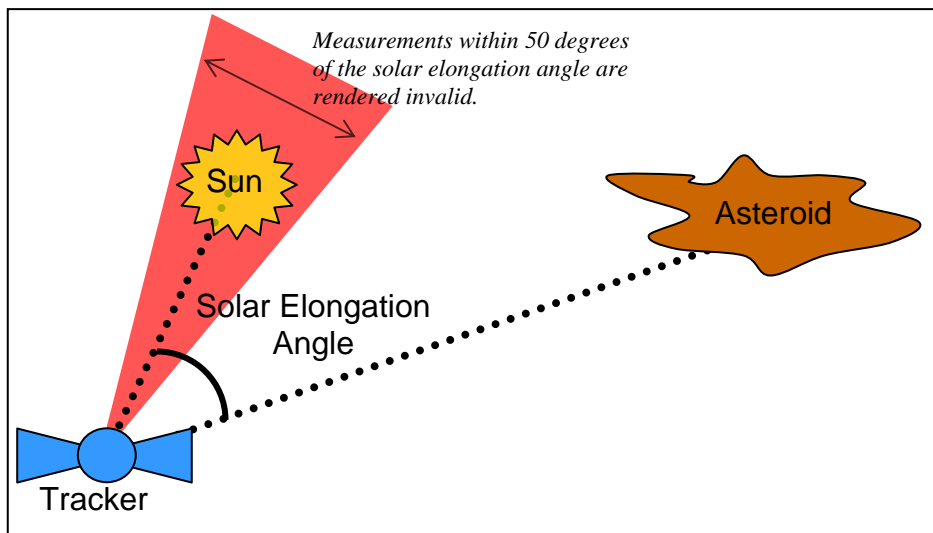


Figure 5 : Definition of the Solar Elongation Angle

C. Visibility Constraint Plots

The following plots (Figures 6 and 7), based on data provided by (Ref. 7), show the solar elongation values and apparent magnitude between June 14, 2006 and March 10, 2009 for which the covariance analysis is performed. The threshold values along with the masking criteria are shown on the plots as well. The bar at the bottom of the graph indicates visibility or no visibility based on both the solar elongation value and the apparent magnitude. The percentage of time that Apophis can be seen from the tracking location during the 1000-day period is noted on the top right hand corner of each plot. The plots shown indicate that the near-Venus orbit tracker has the advantage in terms of visibility criteria and can generate measurements of Apophis more than twice the Earth tracker. However, since this analysis is performed for a single NEO, a general conclusion cannot be drawn (in terms of the visibility constraint) until the complete database of known NEOs is processed.

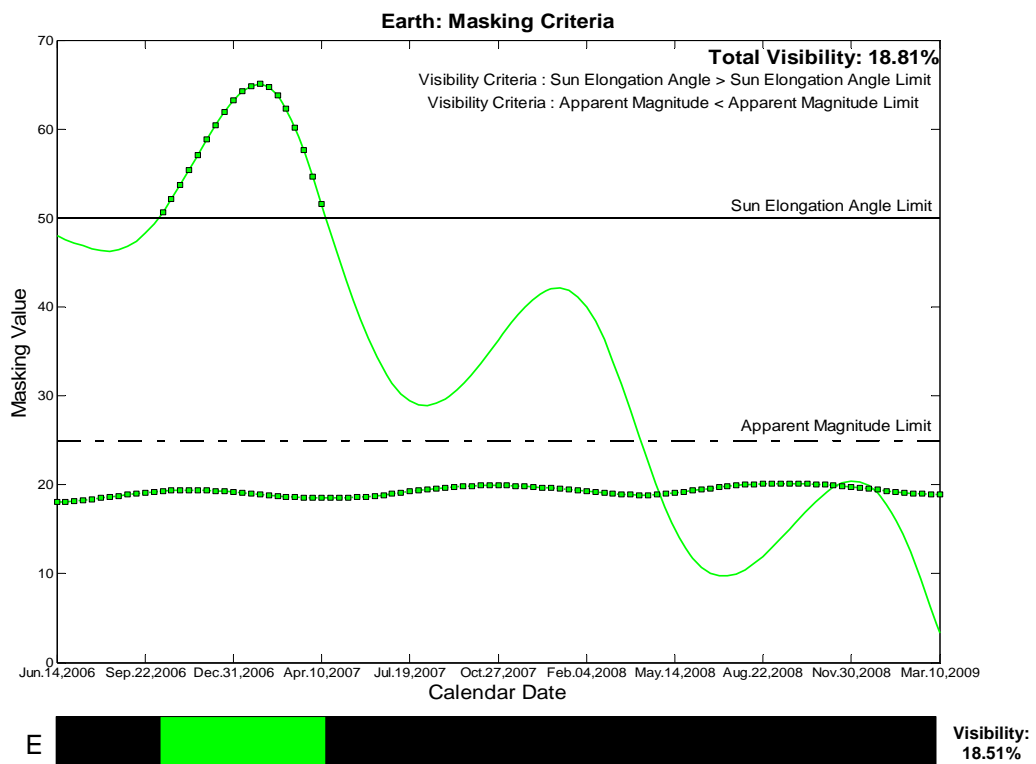


Figure 6 : Earth Masking Criteria

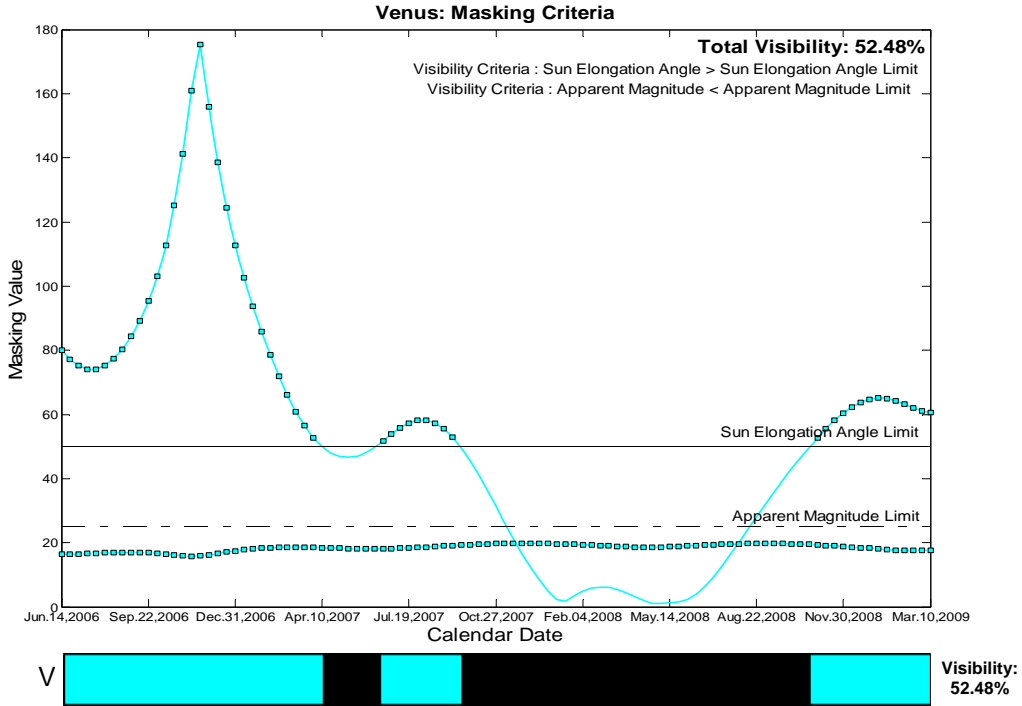


Figure 7 : Venus Masking Criteria

VIII. Analysis Cases

Four different cases are analyzed using combinations among the two tracking locations. For one of the cases, measurements from different tracking stations are combined for the covariance analysis to assess whether using multiple tracking locations is advantageous in terms of reducing position uncertainty. Table 9 below defines the cases that are analyzed.

Table 9 : Analysis Cases

Case/Tracker	Earth (Optical)	Earth (Radar)	Venus (Optical)
Case 1	●		
Case 2	●	●	
Case 3			●
Case 4	●		●

Case 1 assumes that the ground station has only optical measurement capability, so that only right ascension and declination measurements are obtained for the covariance analysis. In Case 2, the ground station is capable of taking right ascension, declination, and range measurements. The purpose of Case 2 is to determine the affect of range measurements on reducing the uncertainty in an object’s position. Case 3 uses optical measurements from the near-Venus orbit tracker, with optical measurements from the Earth ground station added for Case 4. Note that the Earth station in Case 4 does not have radar capability, and only allows for right ascension and declination measurements.

IX. TRACE Block Diagram

To automate visibility masking, external functions are developed to post process the TRACE measurements and generate the masking data and filter out measurements that do not meet the visibility criteria. The following block diagram (Figure 8) outlines the procedure with which the covariance analysis and final results are obtained.

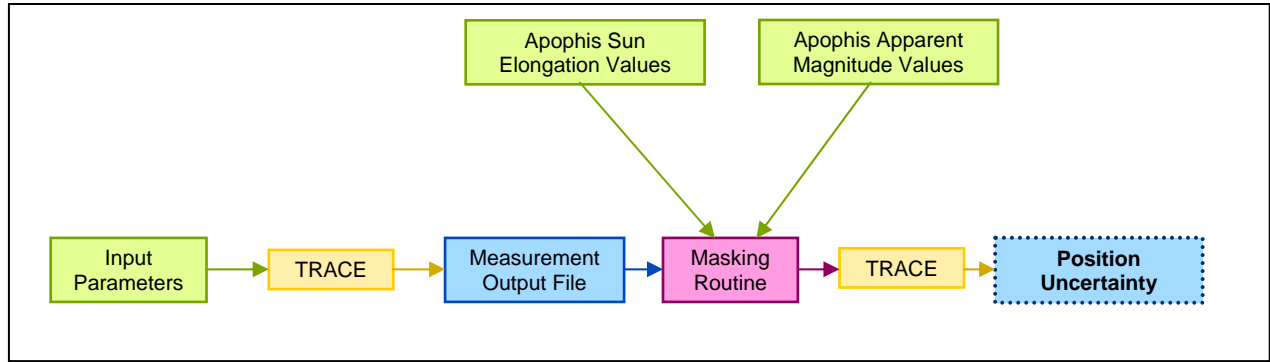


Figure 8 : TRACE Analysis Procedure Block Diagram

X. Results and Discussion

The result of each run consists of the Root Sum Squares (RSS) of the orbit state position 1-sigma uncertainty as a function of time. It is an indication of how accurately the orbit can potentially be determined in the presence of the errors in the estimate of the state as well as the errors in the measurement biases. In the real-time covariance analysis, the general trend of the RSS plot is that the orbit error will decrease as more measurements are processed. When the data stream is interrupted, the error will increase until further data becomes available to again reduce that error.

The results for all four cases are shown in Figures 9 to 12. There are three plots for each figure, with the bottom plot being the enlarged version (in the vertical scale) showing the details of the RSS. The availability of the data is also shown as a colored bar underneath the plots. Figure 9 (Case 1) shows the OD accuracy (for Apophis) that can be obtained with a tracking station placed on the Earth collecting angle measurements only. Figure 10 (Case 2) shows the result from the same station with range measurements in addition to the angle measurements. The similarity between the two results is that when data becomes available, it takes fewer samples in order to reduce the orbit errors to below 100 km. Figure 10 shows that, with additional range measurements, the orbit errors can be reduced further and even more quickly than just having angle measurements. This shows the strength of the radar measurements as valuable data in the OD process even though as a matter of practical purpose, radar measurements may not always be available if, for instance, the object to be observed is too far away for the reflected radar signal to be detected. For this reason, the four remaining cases show results with angle measurements only.

When we place the tracking station near Venus' orbit, Case 3 (Figure 11), the amount of data increases, and the data become more evenly distributed throughout the observation time span. As such, the orbit error can be reduced more quickly when compared to placing the station on Earth. For example, we see that the RSS of the position error is reduced to approximately 50 km, as compared to 100 km in Case 1 (Figure 9), and stays at that level for the entire observation time span even with data outages. Furthermore, by combining additional data from the Earth ground station, Case 4 (Figure 12), it appears that the Apophis orbit can be determined to better than 20 km in roughly the same amount of time; and the error stays below 40 km for the entire 1000 days.

XI. Summary

The results of this study suggest that the location of the tracking station is important because it affects the amount and the distribution of the observations. The quality of the measurements is also important. If the real measurements can be of the same quality as what we have simulated in this study, then the orbit accuracies for real NEO observations could be comparable. The frequency of the observation is the one parameter we do not vary in this study. This of course will change depending on, among other things, the orbital period of the object to be detected. These results are preliminary. In the interest of time, we make use of our existing tool, TRACE, without major modifications and/or enhancements. As such, this study is only a stepping stone for a more rigorous analysis in the near future. Furthermore, this study is focused on only one asteroid, Apophis. A large asteroid population comprising different sizes, shapes, compositions, and orbital periods must be studied to draw a more general conclusion.

Apophis Case 1

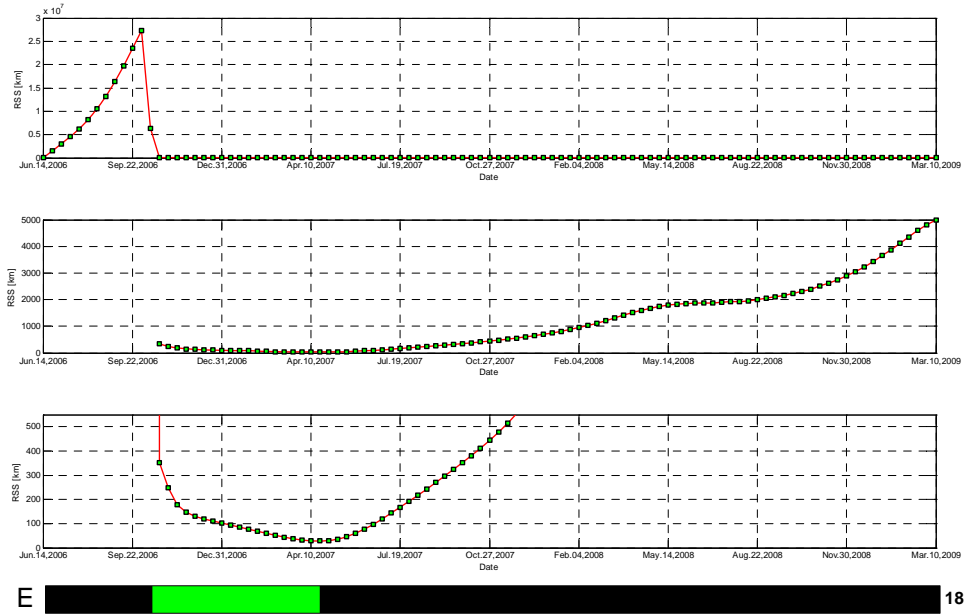


Figure 9 : Case 1, Angle Tracking from Earth

Apophis Case 2

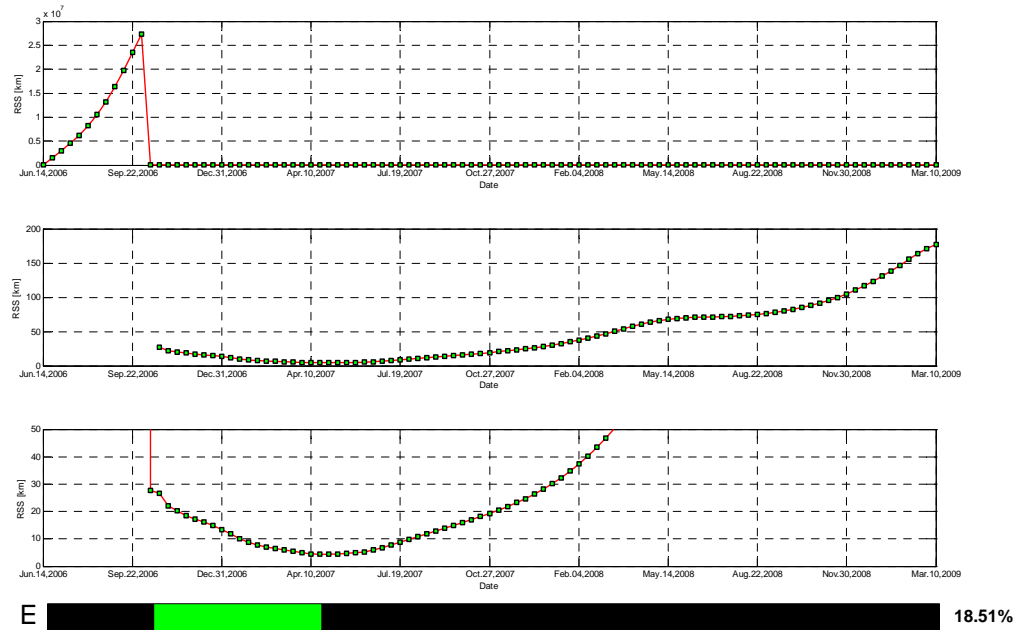


Figure 10 : Case 2, Angle + Range Tracking from Earth

Apophis Case 3

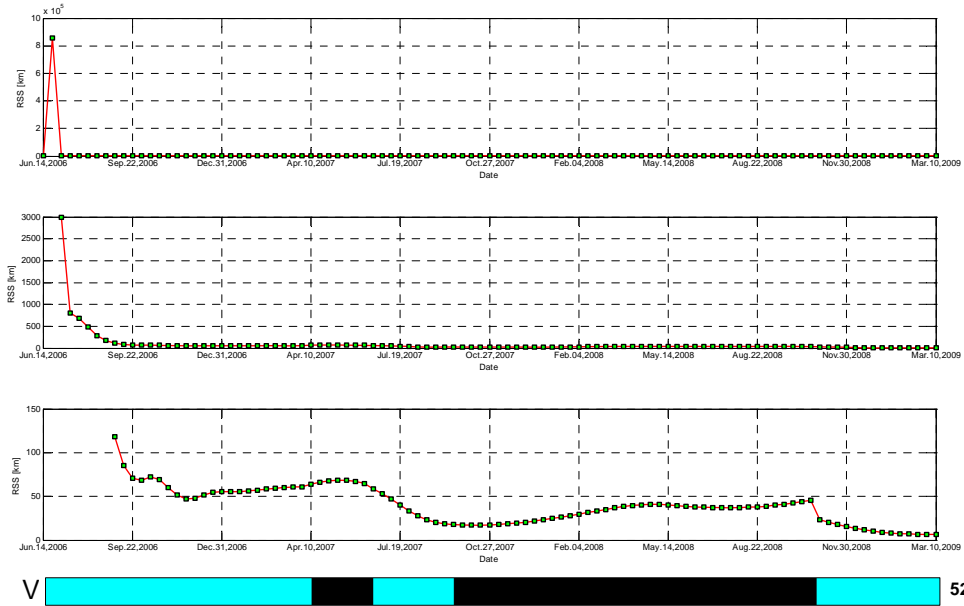


Figure 11: Case 3, Angle Tracking from Near-Venus Orbit

Apophis Case 4

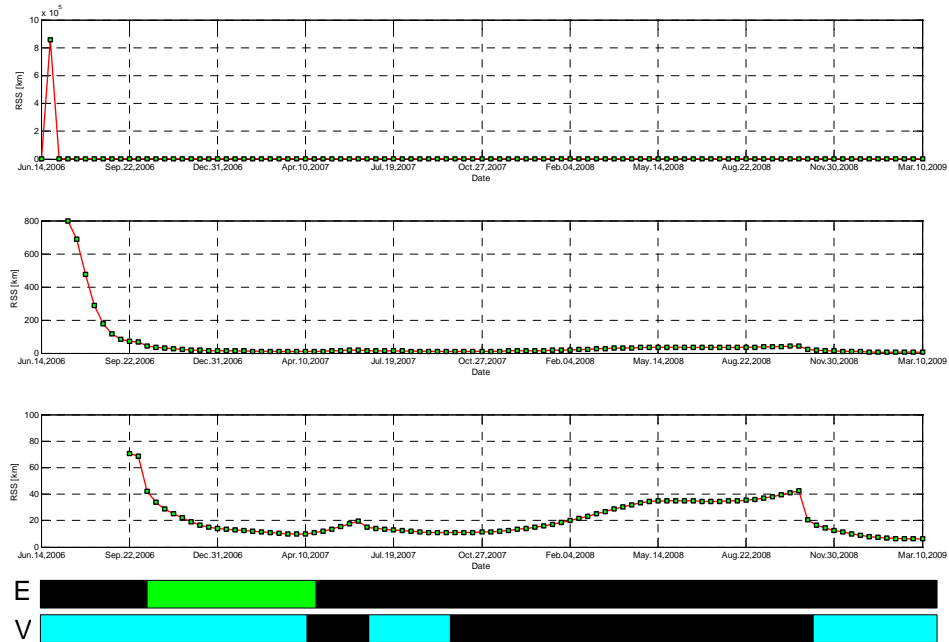


Figure 12 : Case 4, Angle Tracking from Near-Venus Orbit and from Earth

Acknowledgments

We thank M. J. Hart for providing the funding for this study; G. E. Peterson for providing the masking data; E. T. Campbell, J. Y. Cruz, T. E. Gallini, and G. E. Peterson for their constructive comments; and M. J. Wheaton for the travel support to the conference. Some figures are based on images generated by Satellite Orbit Analysis Program (SOAP).

References

- ¹Prislin, R. H., and Downs III, W. D., "TRACE Trajectory Analysis and Orbit Determination Program Vol. I: General Objectives, Description, and Summary," SAMSO-TR-71-141, The Aerospace Corporation, El Segundo, CA, August 1971.
- ²Vallado, D. A., *Fundamentals of Astrodynamics and Applications*, McGraw-Hill, New York, 1997, Chap. 9.
- ³Chamberlin, A., "NEO Discovery Statistics," *Near Earth Object Program* [online database], URL: <http://neo.jpl.nasa.gov/stats> [cited 21 February 2007].
- ⁴Yeomans, D., "99942 Apophis (2004 MN4) Earth Impact Table," *Near Earth Object Program* [online database], URL: <http://www2.jpl.nasa.gov/neo/risk/a99942.html> [cited 21 February 2007].
- ⁵Hart, M. J., Private Communication, September, 2006.
- ⁶McCarthy, D. D., "IERS Conventions (1996)," U.S. Naval Observatory., IERS Technical Note 21, Washington, DC. 1996.
- ⁷Peterson, G. E., Private Communication, September, 2006.