The 2007 Planetary Defense Conference
March 5-8, 2007
The George Washington University in Washington, D.C., USA

An Analysis of the Correction Problem for the Near-Earth Asteroid (99942) Apophis=2004 MN4

*V.V. Ivashkin * and C.A. Stikhno ***

*** Keldysh Institute of Applied Mathematics, Moscow, Russia**

****Lavochkin Association, Khimki, Russia**

INTRODUCTION

A problem of the orbit correction for the asteroid Apophis is analyzed in the paper.

This correction has to prevent the asteroid-Earth collision in 2036.

Now there is any probability of this collision. So, it is important to analyze possible prevention of this collision as well as to determine better the Apophis orbit and the Apophis fly-by near the Earth in 2029 and following flight.

Contents

1. Nominal Trajectory of Asteroid Apophis

a) A Model and the Equations of the Asteroid Motion

$$\ddot{\mathbf{r}} = -\frac{\mu_S}{|\mathbf{r}|^3}\mathbf{r} - \sum_i \mu_i \left(\frac{\mathbf{r}_i}{|\mathbf{r}_i|^3} + \frac{\mathbf{r}-\mathbf{r}_i}{|\mathbf{r}-\mathbf{r}_i|^3} \right) + \Delta_1 + \Delta_2 \qquad \textbf{(1)}$$

r, $\mathbf{r_i}$ – asteroid's and celestial bodies' radius-vectors (planets and the Moon – from DE-405);

μ_S – Gravitational parameter of the Sun;

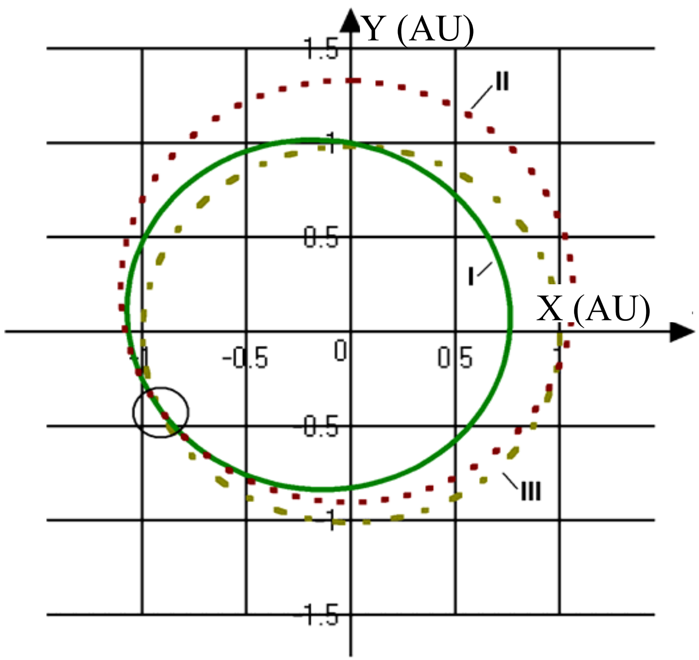
μ_i – Gravitational parameters of celestial bodies;

Δ_1 – takes into account the Earth oblateness;

Δ_2 – takes into account the solar-radiation pressure.

Set aside: small planets, relativistic effect and the Yarcovsky effect

1. Nominal Trajectory of Asteroid Apophis
b) Characteristics of *Apophis*



- **Fig. 1. Orbits of Apophis and Earth**
- **I – ** Asteroid's orbit before approaching the Earth in 2029
- **II** –Nominal Trajectory of *Apophis* after fly-by the Earth in 2029
- **III** – Orbit of the Earth

Apophis' orbit **I** :

- sidereal period T=0.89 y. (synodic ≈8 years)
- aphelion R_α=1.10 AU
- perihelion R_π=0.75 AU
- major semiaxis a=0.92 AU; incl. I=3°.3

Apophis' orbit **II** (nominal):

- sidereal period T=1.17 y. (synodic ≈ 7 years)
- aphelion R_α =1.33 AU
- perihelion R_π =0.90 AU
- major semiaxis a=1.11 AU; incl. I=2°.1

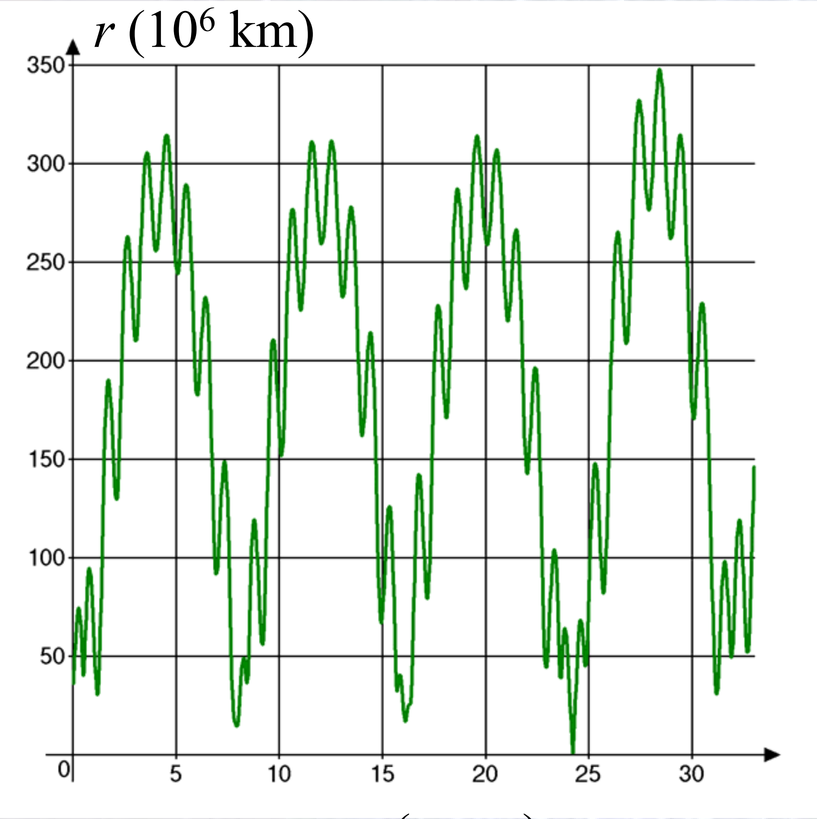
Geocentric Velocity "at infinity" ≈5.5 km/sec

Collision velocity ≈12.6 km/sec

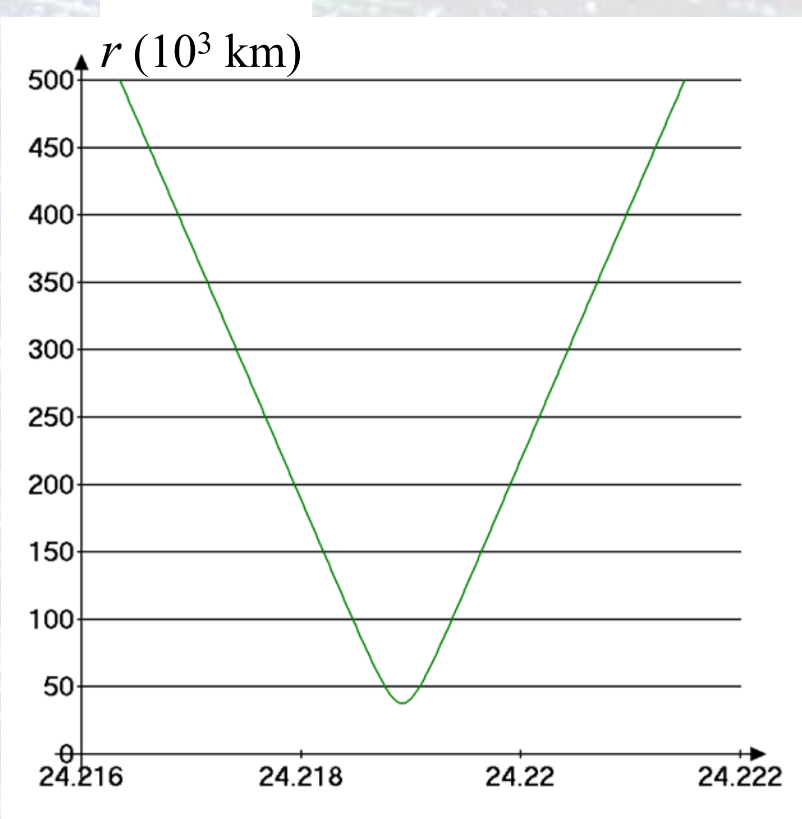
Physical characteristics:

- Diameter D_A=250-390 m
- Density d_A=2.5-3 g/cm^3
- Mass (D_A=320 m, d_A=2.5 g/cm^3) $m_A \approx 4.3 \cdot 10^{10}$ kg
- Energy of Collision with the Earth ~ 800 MT TNT

I. Nominal Trajectory of Asteroid Apophis
c) Asteroid-Earth Distance



t-t_0 (years)



t-t_0 (years)

Figure 2. Asteroid's nominal distance to Earth in 2005-2038
(t - t_0 is the time relatively t_0 = 2005 JAN. 30.0)

Figure 3. Asteroid's distance to Earth for approaching in 2029
(X is the time relatively t_0 = 2005 JAN. 30.0)

2. "Tube" of Asteroid's Trajectories

Confidence set *D* of asteroid's kinematic parameters at initial moment t_0=2005 JAN. 30.0 according to 1σ criterion:

$$D=\{ |\Delta r_0| \le 3 \text{ km}; \ |\Delta V_0| \le 2 \text{ m/sec} \} \qquad (2)$$

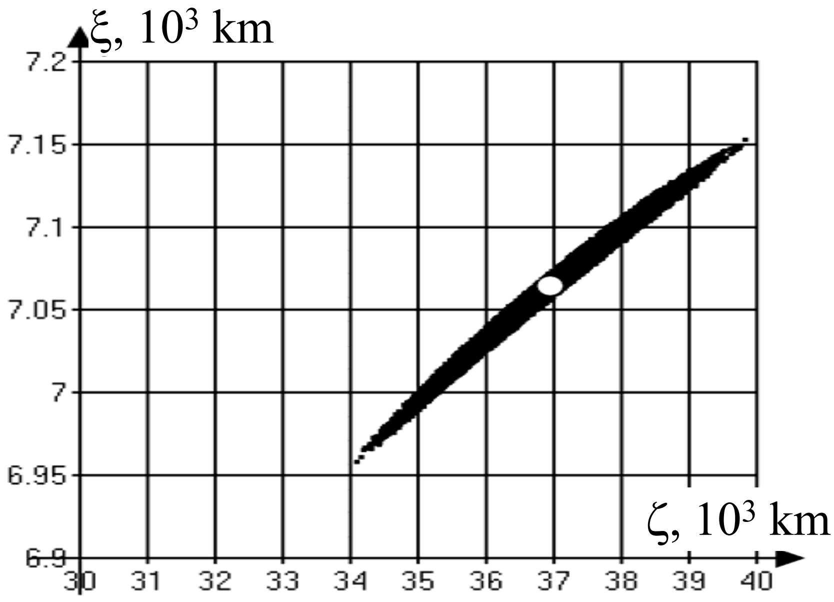


Figure 4. The "dispersion ellipse" for Asteroid's trajectories in the aim plane near the Earth in April 2029 :
$\zeta_0 \approx$ 36 948 km, $\Delta\zeta \approx$3000 km
$\xi_0 \approx$ 7 064 km, $\Delta\xi \approx$ 100 km

Parameters of Approaching in 2029:
$r_\pi \approx$ 37600 +/- 3000 km; $t_f = t_{f\,n}$ +/- 120 sec

3. A Set of Trajectories for Asteroid-Earth Collision in 2036 - a

Search for trajectories hitting the Earth

1) For initial state vectors Z=($\mathbf{r_0}$,$\mathbf{V_0}$) from previous set D, we consider a function

$$f(Z)=r_{\pi}\,(Z,\, t_{\pi \mathrm{i}})\ , \qquad \textbf{(3)}$$

which gives the value of minimal distance between asteroid and the Earth for some moment $t_{\pi \mathrm{i}}$ of close approach with the Earth;

2) For a set of 10^4 initial vectors $Z \in$D, we calculate $r_{\pi}(Z)$ for all moments of approach

3) Determine a set of «dangerous» trajectories with $r_{\pi} < 10^{6}$ km. Then, taking them as starting conditions, we compute trajectories, which achieve local minimum for function $r_{\pi}(Z)$ using a gradient minimization method:

$$Z_{\mathrm{k+1}} = Z_{\mathrm{k}} - \theta_{\mathrm{k}}\, \mathbf{grad}\, f(Z_{\mathrm{k}})\ , \qquad \textbf{(4)}$$

where θ_{k} is a step along antigradient direction for the function $f=r_{\pi}(Z)$.

By this way, we found more then **2000 dangerous** trajectories (**$r_{\pi} < 10^{6}$ km**) for a time interval 2036-2113, including 131 ones for April 2036. This gives **a set of hitting trajectories with the Earth-Apophis collision in 2036 (r_{π} <6371 km).**

3. A Set of Trajectories for Asteroid-Earth Collision in 2036 - b

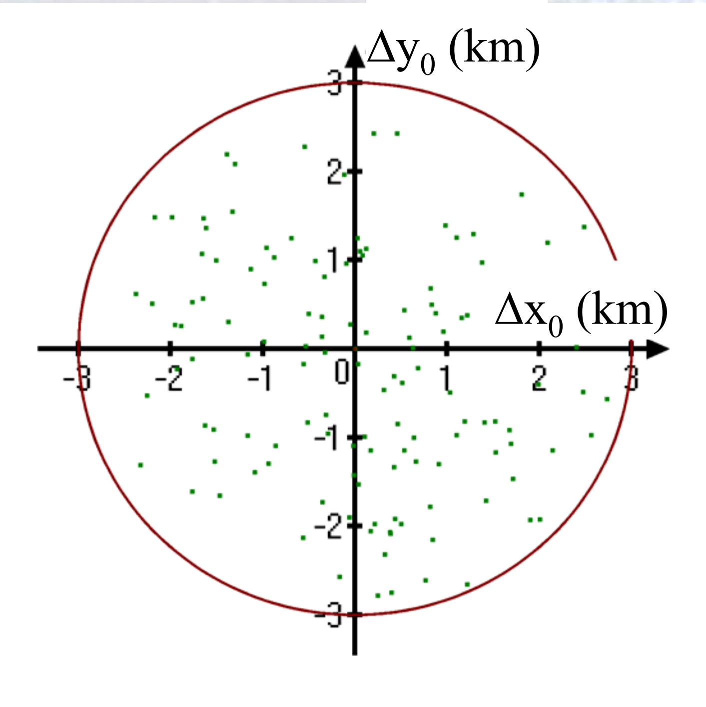


Figure 5. Variations of initial coordinates X, Y which correspond to the collision trajectories in April 2036 (on plane XY)

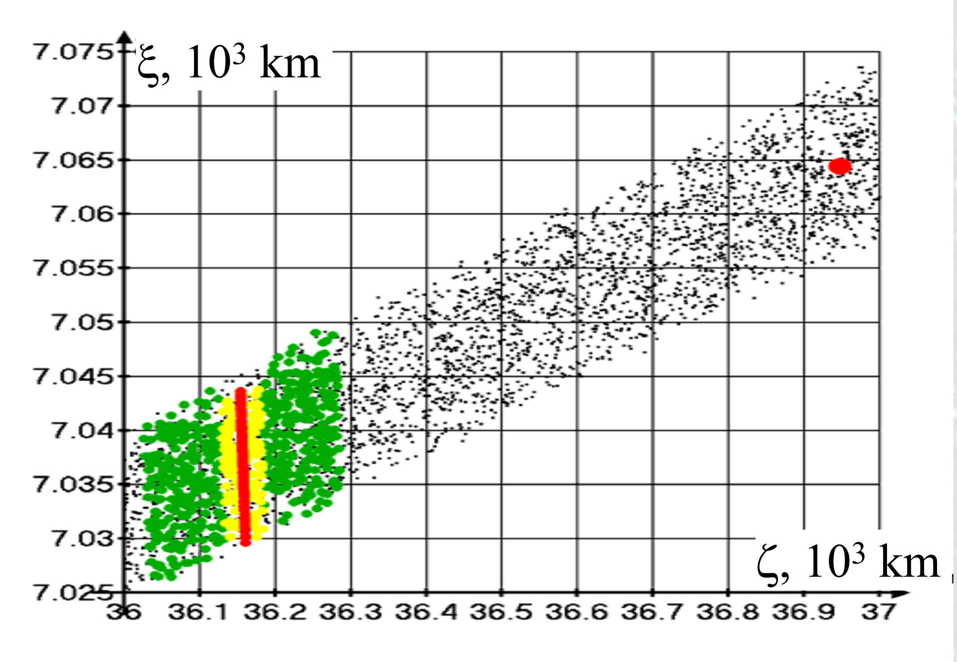


Figure 6. Sets of "dangerous" and collision trajectories (in April 2036) on the aim plane near Earth in 2029:

- **narrow red strip, r_π<6371 km,** ζ ~36160 km**;**
- **surrounding yellow band, r_π<1*10⁶ km**;
- **green band, r_π<5*10⁶ km**, ζ~36000-36300 km

3. A Set of Trajectories for Asteroid-Earth Collision in 2036 - c

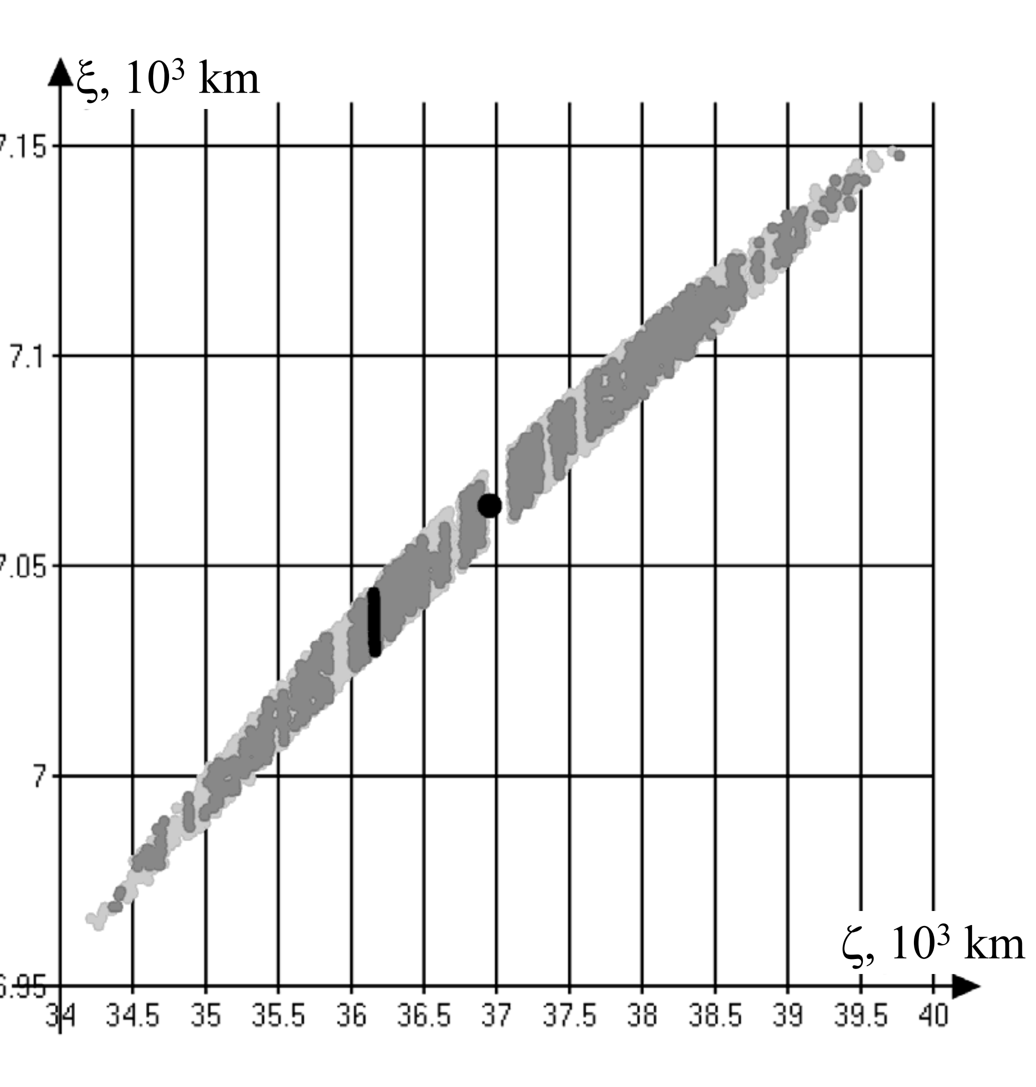


Figure 7. Set of trajectories (in the aim plane near Earth in 2029), which have at least one close approach with the Earth from year 2036 till 2113:

A) Black small disk (the nominal orbit) and white surrounding band have no close approach;

B) Light bands have r_π =1-5 mln km;

C) Dark domain has r_π =6371 km 1 mln km;

D) Black line corresponds to hitting trajectories with r_π< 6371 km in 2036.

4. Correction of Asteroid's Orbit-a

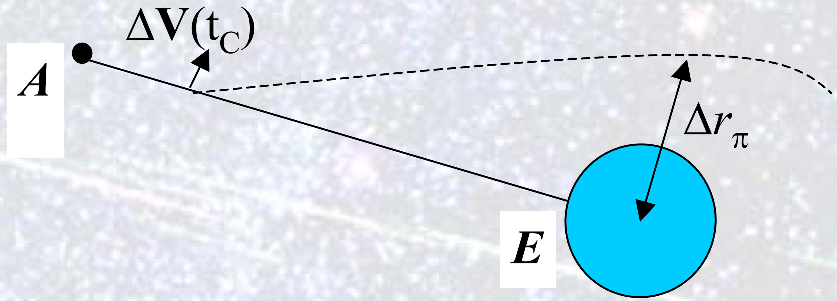


Figure 8. Scheme of one-impulse correction for a collision trajectory

Schemes of Asteroid's Orbit Correction

1. One-impulse correction

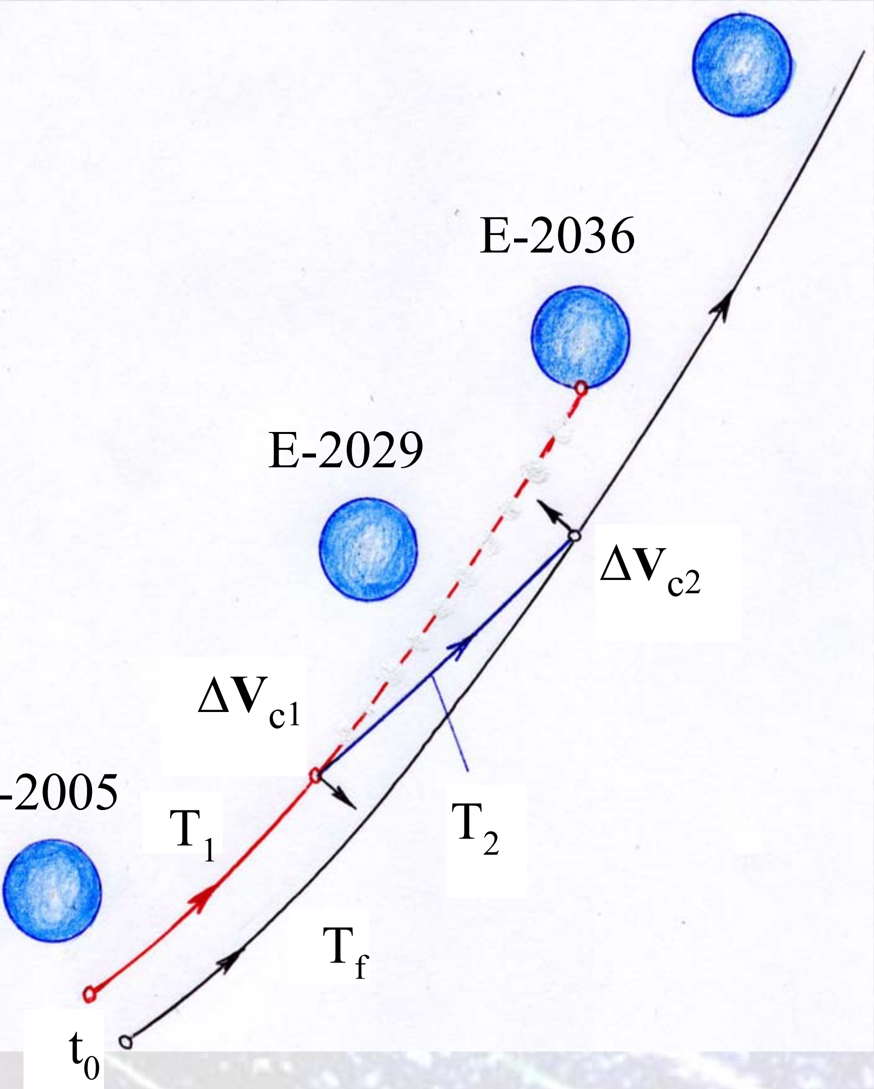
One-parameter correction of perigee distance for 2036;

One-parameter correction of flyby point distance on aim plane near Earth in 2036;

One-parameter correction of perigee distance for 2029;

Two-parameter correction of flyby point coordinates on aim plane near Earth in 2036.

4. Correction of Asteroid's Orbit-b



2. Two-impulse correction

- **a transfer of asteroid from the collision trajectory (T_1) to a "good" enough one (T_f), which has no close approaching with the Earth for next hundred years**

Figure 9. Scheme of two-impulse correction of the hitting trajectory

4. Correction of Asteroid's Orbit-c

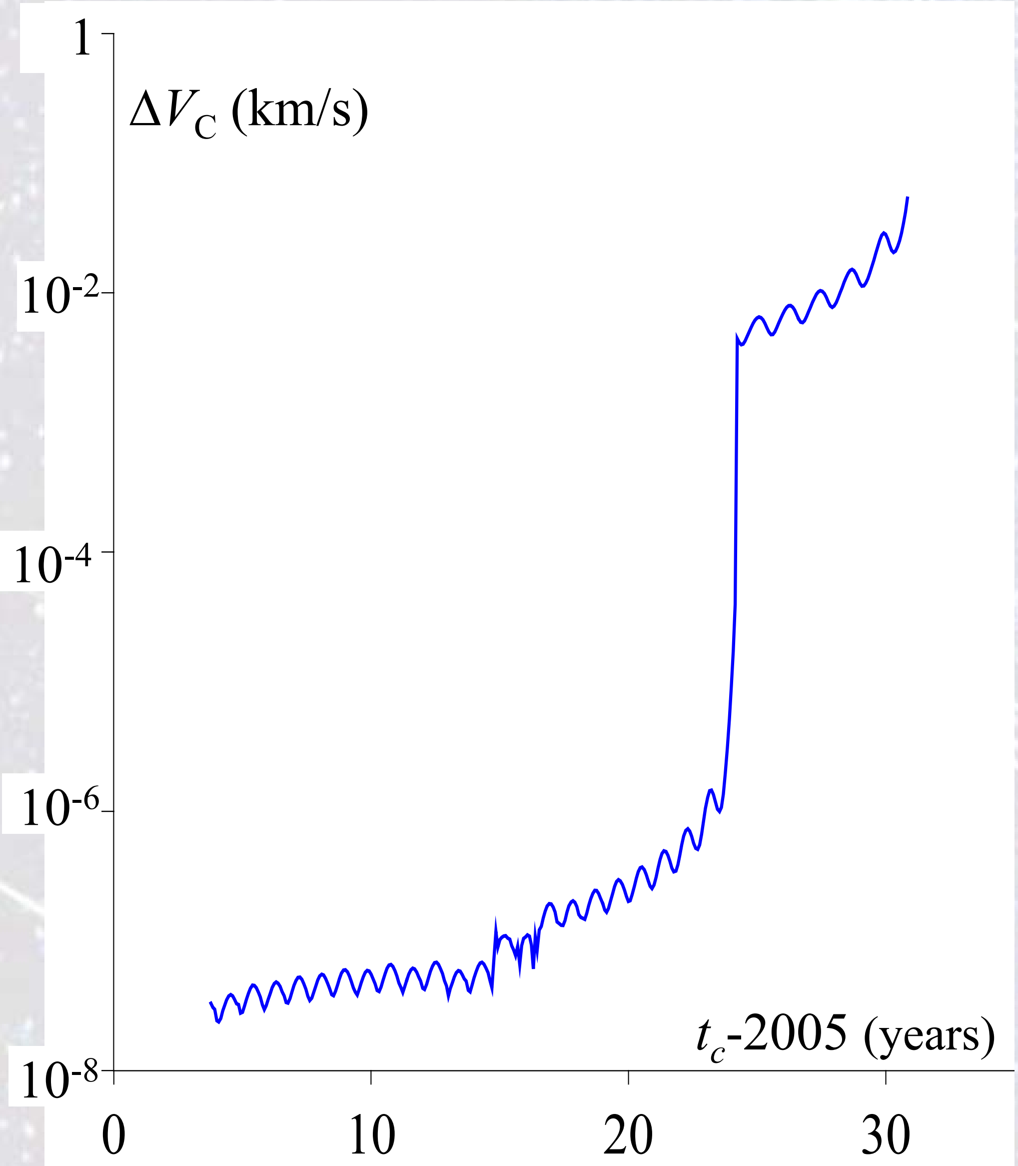


Figure 10. Value of Correction Velocity Impulse to give deflection of *Apophis* from the Earth $\Delta r_\pi=10^6$ km in April 2036

4. Correction of Asteroid's Orbit – d

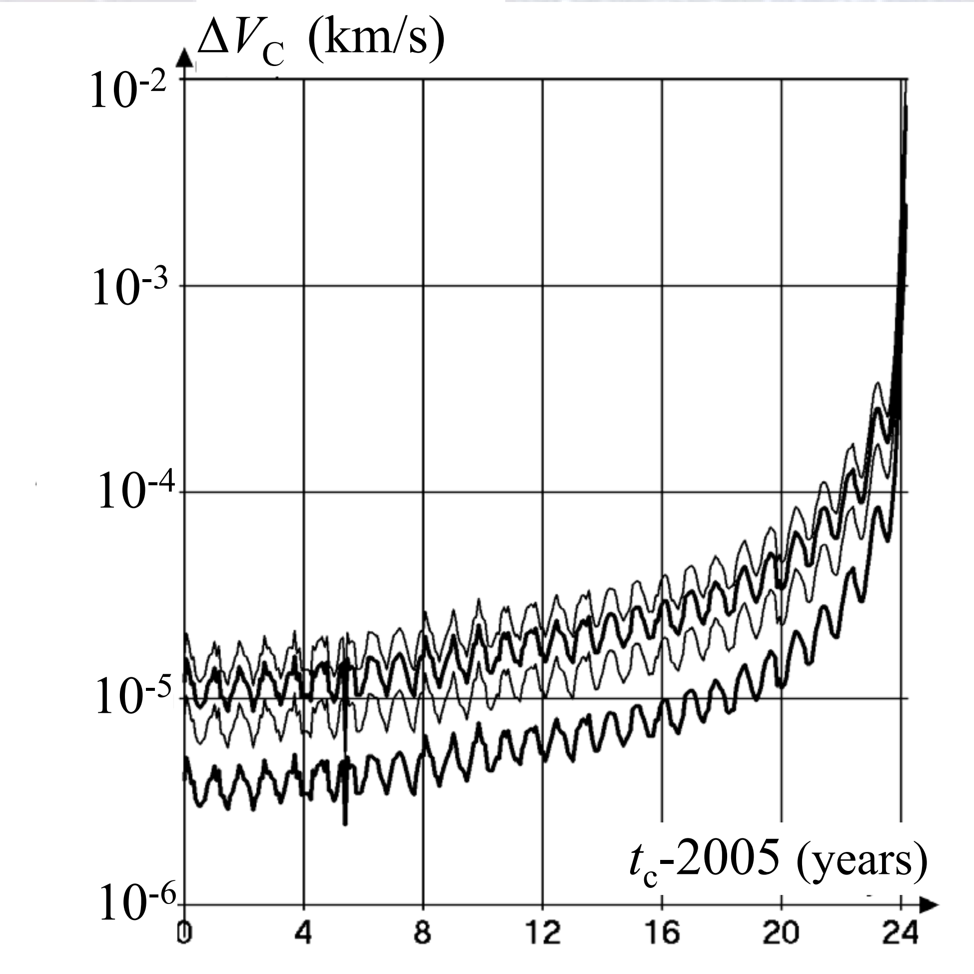


Figure 11. Value of the Correction Velocity Impulse to give deflection of *Apophis* from the Earth Δr_{π}=(5, 10, 15 и 20) $*10^3$ km in April 2029.

4. Correction of Asteroid's Orbit – e

Table 1. Spacecraft mass m_{SC} (near Apophis) and m_0 (on LEO) for kinetic-impact effect on Apophis to correct its fly-by near Earth in 2036

t_c, year	Δr_π, km	ΔV_c, m/s	m_{SC}, t A/B	m_0, t A/B
2020-2021	$1*10^6$	0.6 10^{-4}	1 / 0.4	2 / 0.8
2028	$1*10^6$	10^{-3}	14 / 5	31 / 11
2029-	$1*10^6$	10^{-2}	140 / 50	310 / 110
2029^+	$30*10^3$	0.12	1700 /610	3700/1300

A)–Model for perfectly inelastic collision of the asteroid and spacecraft;

B)–the Stanyukovich Model for high-speed explosive collision

4. Correction of Asteroid's Orbit – f

Table 2. Valuation of Energy E_N for thermonuclear effect (surface explosion)

t_C, year	2029-	2029 +	2031	2033	2035 - 1	2035 - 2
Δr_π, km	$1*10^6$	$1*10^6$	$1*10^6$	$1*10^6$	$1*10^6$	$1*10^6$
ΔV_C, m/s	10^{-2}	4	6	8	21	54
E_N, MT [1, 2]	0.004	1.6	2.4	3.2	8.3	22

1. Ahrens T.J., Harris A.W., 1992
2. Nechay V.Z., Nogin V.G., et al., 1997.

5. Conclusions –a

Analysis of the Apophis motion shows that near its nominal orbit, in a tube of possible (for present level of our knowledge for the Apophis motion) orbits, there is a small set of the Apophis orbits which have collisions with the Earth in 2036.

This collision for the impact energy of about 1000 MT TNT could result in big destructions and human victims.

Because of this, it is important to analyze the problem to prevent the Apophis – Earth collision if the hitting orbit of the asteroid is realized.

It is also very important to know better the asteroid orbit and develop International Program for both ground measurements especially during Apophis-Earth close approaching in 2013, 2021 and sending to the Apophis a special spacecraft with some radio-device.

Determination of physical characteristics for Apophis is important, too.

5. Conclusions –b

Authors would like to express our appreciation to **Prof. Shor V.A.** (Institute of Applied Astronomy, St. Petersburg) as well as to

Dr. Bykova L.E. and Dr. Galushina T.Ju. (State University, Tomsk) for very useful cooperation.

The study is supported by by the Grant for the Scientific School NSc-2003.2003.1.

Thank you for attention!